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कक्षा - 12

Chapter - 7

समाकलन

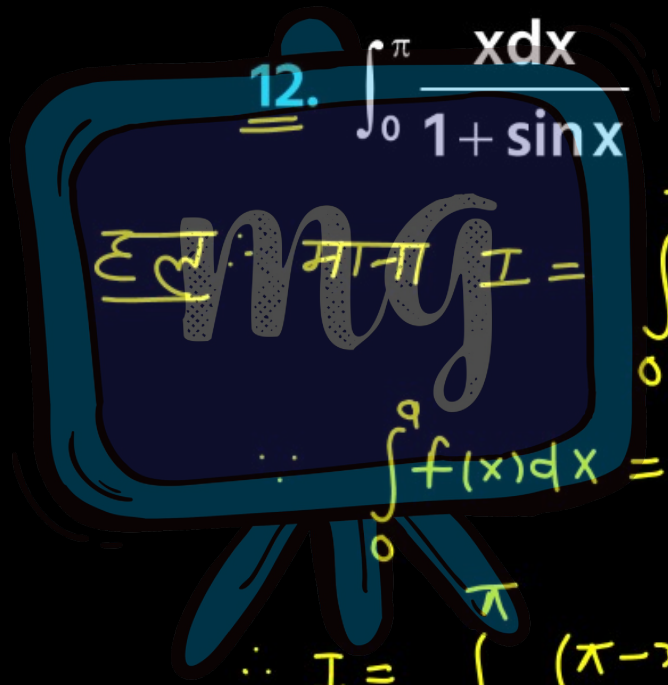
निश्चित समाकलनों के गुणधर्म

प्रश्नावली 7.10

Part - 21

Lakhan Saini

प्रश्नावली 7.10



12. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$

हल : माना $I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$ — (1)

$\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\therefore I = \int_0^{\pi} \frac{(\pi-x)\,dx}{1+\sin(\pi-x)}$

II

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$$I = \int_0^{\pi} \frac{\pi}{1+\sin x}\,dx - \int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

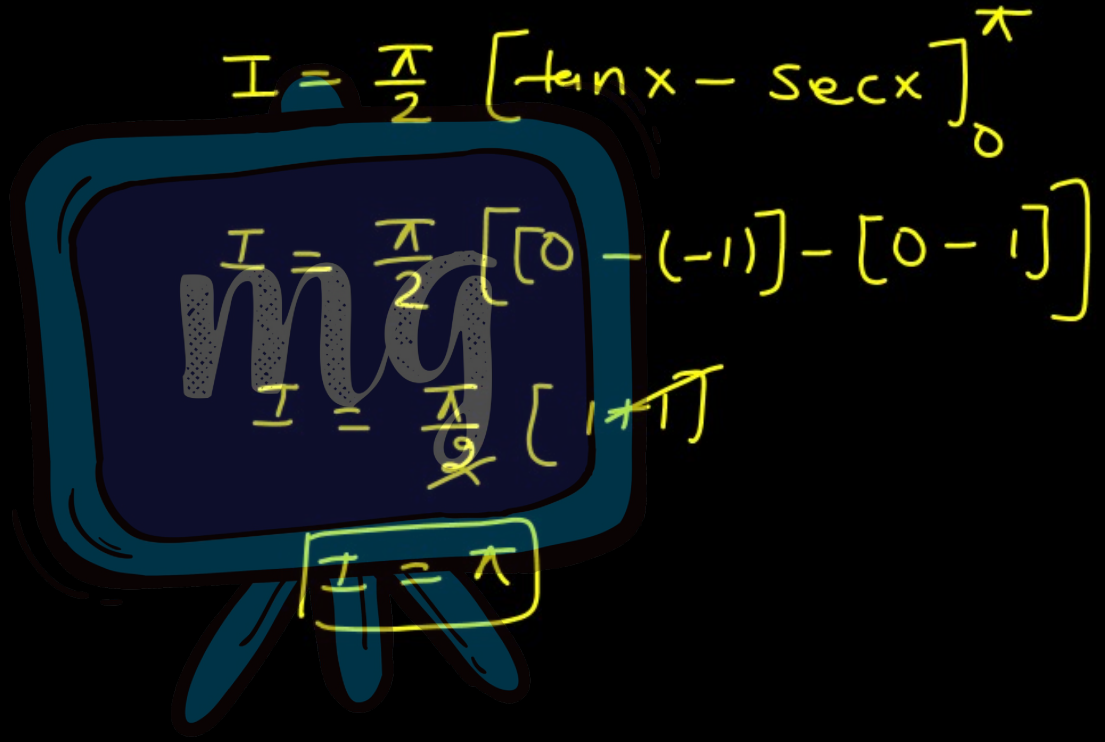
$$I = \pi \int_0^{\pi} \frac{1}{1+\sin x}\,dx - I$$

$$2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{1-\sin x}{\cos^2 x}\,dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} (\sec^2 x - \sec x \tan x)\,dx$$

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$$2I = 2\pi \int_0^{\pi/2} \frac{1}{1+\sin x}\, dx$$

$$I = \pi \int_0^{\pi/2} \frac{1-\sin x}{\cos^2 x}\, dx$$

$$I = \pi \int_0^{\pi/2} (\sec^2 x - \sec x \tan x)\, dx$$

$$= \pi \left[\tan x - \sec x\right]_0^{\pi/2}$$

$$= \pi \left[\right.$$

प्रश्नावली 7.10

13. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$

हल :-

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx \\ \text{जब } f \to \text{समफलन} \\ 0, f \text{ विषम फलन} \end{cases}$$

$$I = \int_{-\pi/2}^{\pi/2} \sin^7 x\,dx$$

$$[\sin(-x)]^7 = -\sin^7 x$$

$$I = 0$$

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14. $\int_0^{2\pi} \cos^5 x\,dx$

हल :- $I = \int_0^{2\pi} \cos^5 x\,dx$

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

जब $f(x) = f(2a-x)$

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$$

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$\because \quad \cos^5 x = (\cos(2\pi - x))^5$

$\therefore \quad \int_0^{2\pi} \cos^5 x \, dx = 2\int_0^{\pi} \cos^5 x \, dx$

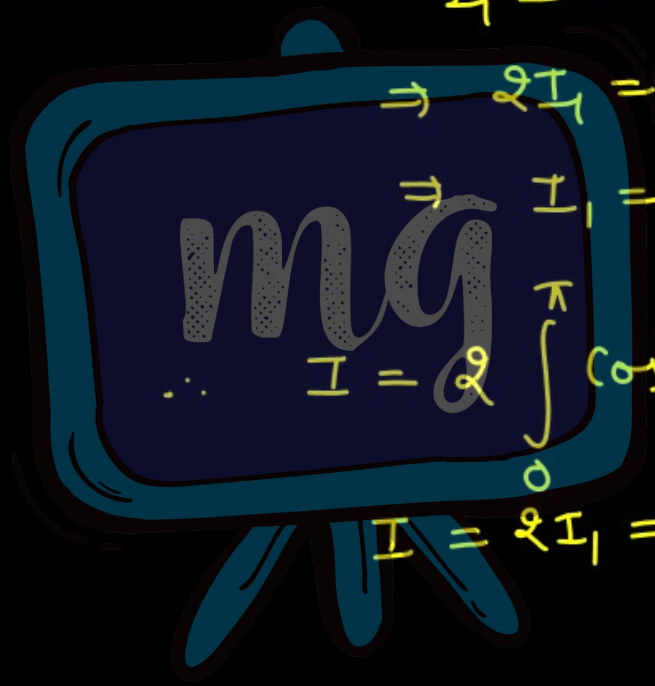
मानो $I_1 = \int_0^{\pi} \cos^5 x \, dx$ — (1)

$I_1 = \int_0^{\pi} (\cos(\pi - x))^5 \, dx$

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I_1 = -\int_0^{\pi} \cos^5 x \, dx$

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$$I_1 = -I_1$$

$$\Rightarrow 2I_1 = 0$$

$$\Rightarrow I_1 = 0$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x \, dx$$

$$I = 2I_1 = 2(0) = 0$$

प्रश्नावली 7.10

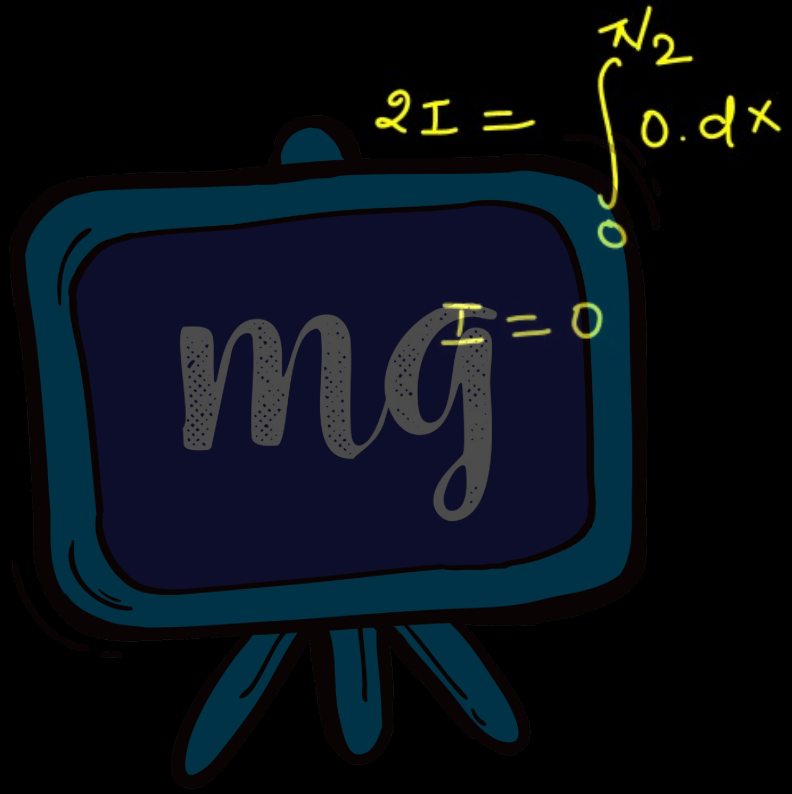
15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

हल :- माना $I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ — ①

$\because \int_0^a f(x) dx = \int_0^a f(a-x) dx$

$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$ — ②

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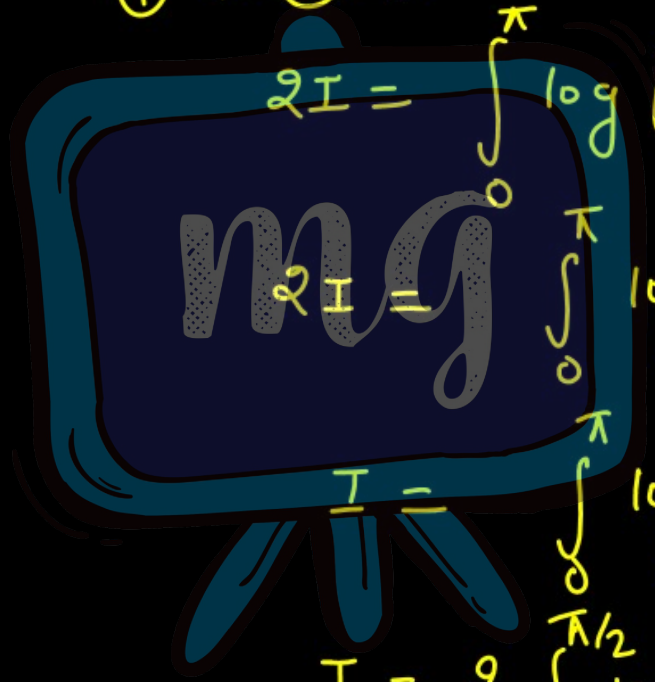
16. $\int_0^{\pi} \log(1+\cos x)dx$

हल :- माना $I = \int_0^{\pi} \log(1+\cos x)dx$ —①

$I = \int_0^{\pi} \log[1+\cos(\pi-x)]dx$ $\left\{\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right.$

$I = \int_0^{\pi} \log(1-\cos x)dx$ —②

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① + ② से -

$$2I = \int_0^{\pi} \log\left[1-\cos^2 x\right] dx$$

$$2I = \int_0^{\pi} \log \sin^2 x \, dx$$

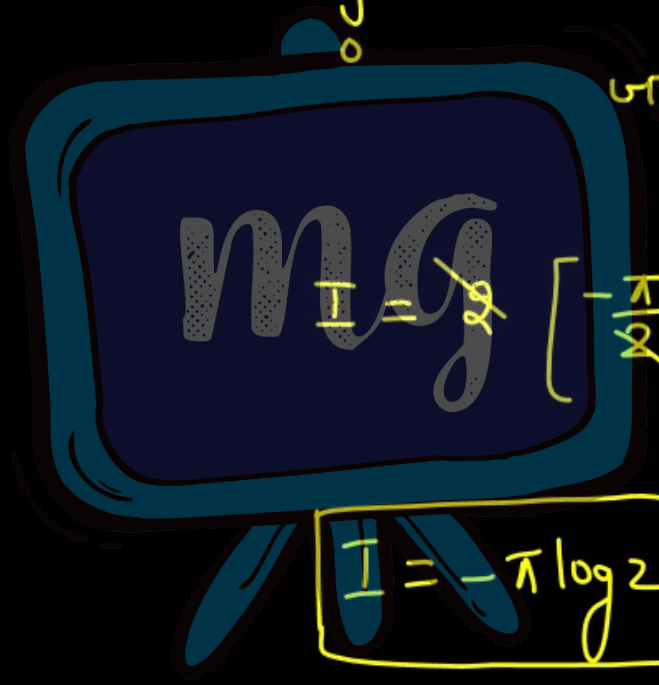
$$I = \int_0^{\pi} \log \sin x \, dx$$

$$I = 2\int_0^{\pi/2} \log \sin x \, dx \quad \left[\because \log \sin x = \log \sin(\pi - x)\right]$$

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$$\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$$

जब $f(x) = f(2a-x)$



$$I = \cancel{2}\left[-\frac{\pi}{\cancel{2}}\log 2\right]$$

$$\left\{\because \int_0^{\pi/2} \log\sin x\,dx = -\frac{\pi}{2}\log 2\right.$$

$$\boxed{I = -\pi \log 2}$$

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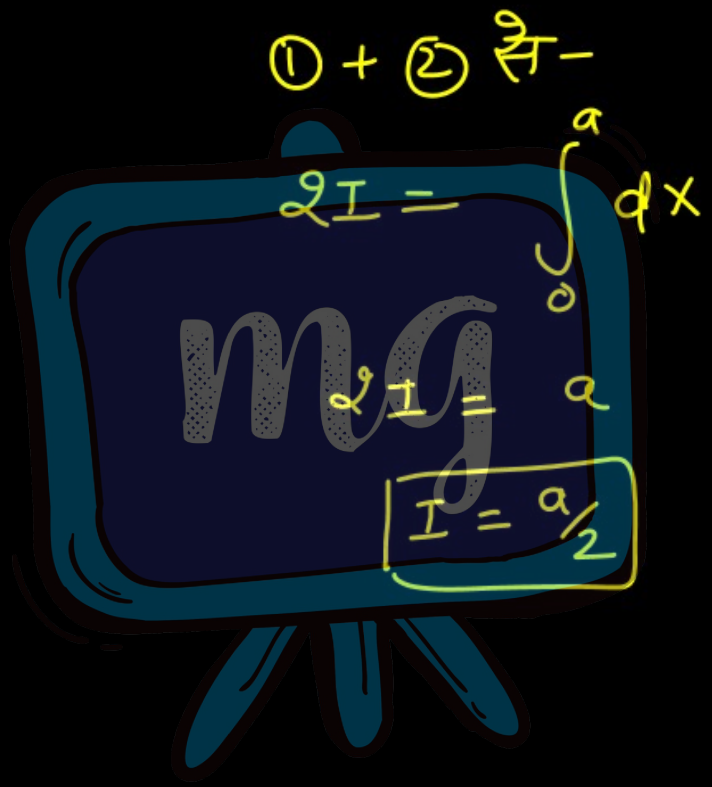
17. $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

हल :- $\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx \quad \text{—— (1)}$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx \quad \text{—— (2)}$$

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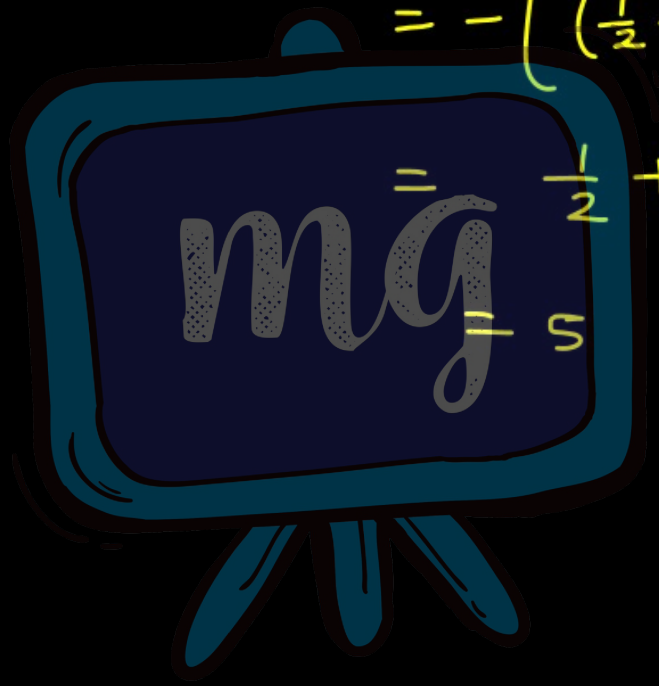
18. $\int_0^4 |x-1|\,dx$

हल :- $|x-1| = \begin{cases} -(x-1) & x < 1 \\ x-1 & x \geq 1 \end{cases}$

$\therefore \quad I = \int_0^4 |x-1|\,dx = -\int_0^1 (x-1)\,dx + \int_1^4 (x-1)\,dx$

$= -\left[\frac{x^2}{2} - x\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$

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$$= -\left[\left(\frac{1}{2}-1\right)-0\right]+\left[(8-4)-\left(\frac{1}{2}-1\right)\right]$$

$$= \frac{1}{2}+4+\frac{1}{2}$$

$$= 5$$

प्रश्नावली 7.10

19. दर्शाइए $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, कि यदि f और g को f(x) = f(a–x) एवं g(x) + g(a – x) = 4 के रूप में परिभाषित किया गया है।

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माना $I = \int_0^a f(x)g(x)dx$ —— (1)

$I = \int_0^a f(a-x)\,g(a-x)dx$ $\left[\int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^a f(x)\,g(a-x)dx$ —— (2)

(1) + (2) से —

$2I = \int_0^a f(x)\left[g(x)+g(a-x)\right]dx$

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$$2I = 4\int_0^a f(x)\,dx$$

$$I = 2\int_0^a f(x)\,dx$$

$$\therefore \int_0^a f(x)g(x)\,dx = 2\int_0^a f(x)\,dx$$

H-P

प्रश्नावली 7.10

प्रश्न 20 एवं 21 में सही उत्तर का चयन कीजिए।

20. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x + 1)dx$ का मान है:

(a) 0

(b) 2

(c) π

(d) 1

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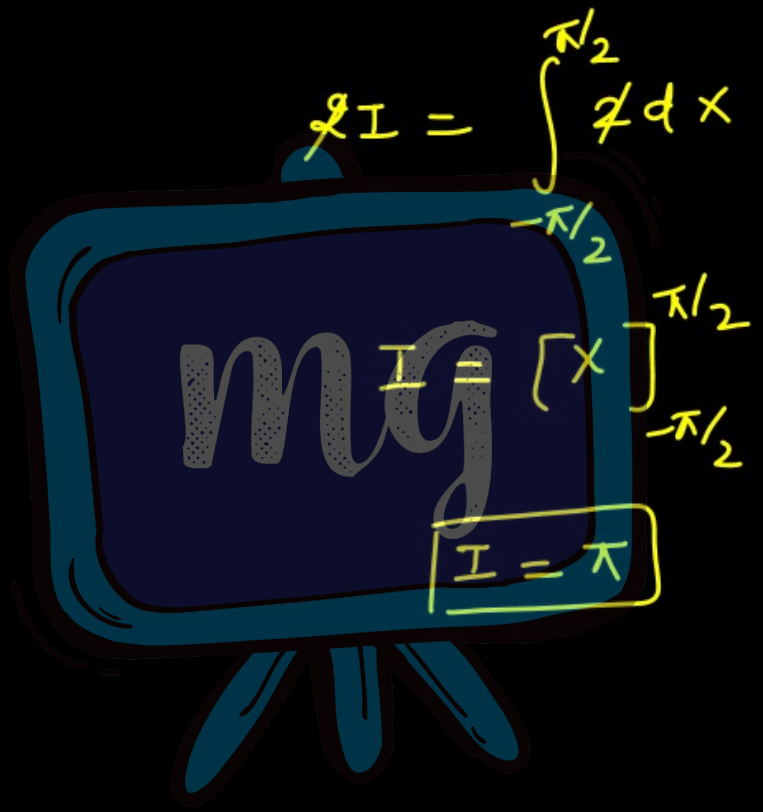
माना $I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\,dx$ — (1)

$I = \int_{-\pi/2}^{\pi/2} [(-x)^3 + (-x)\cos(-x) + [\tan(-x)]^5 + 1]\,dx$

$\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{-\pi/2}^{\pi/2} (-x^3 - x\cos x - \tan^5 x + 1)\,dx$ — (2)

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$\int_0^a f(x)\,dx$ $\quad \int_0^a f(a-x)\,dx$

$\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ का मान है: — (1)

(a) 2

(b) 0

(c) $\frac{3}{4}$

(d) −2

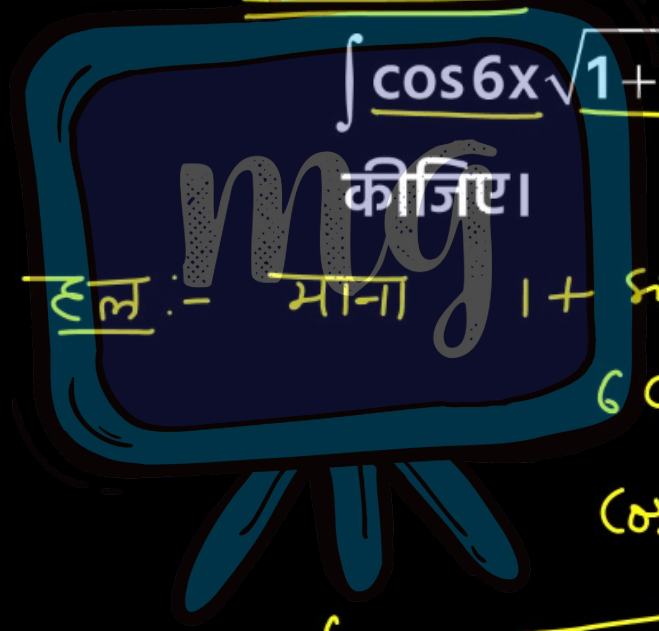
$I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx$ — (2)

$I = \int_0^{\pi/2} 0 \cdot dx = 0$

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उदाहरण-35 :

$\int \cos 6x \sqrt{1+\sin 6x}\,dx$ का मान ज्ञात कीजिए।



हल :- माना $1+\sin 6x = t^2$

$6\cos 6x = 2t\frac{dt}{dx}$

$\cos 6x\,dx = \frac{t}{3}dt$

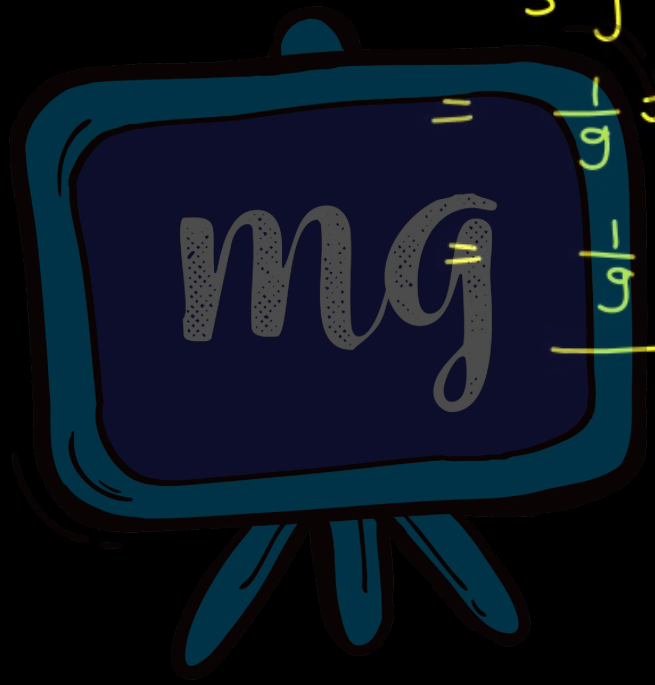
$\therefore \int \cos 6x \sqrt{1+\sin 6x}\,dx = \frac{1}{3}\int \sqrt{t^2}\,t\,dt$

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$$= \frac{1}{3}\int t^2\,du$$

$$= \frac{1}{9}t^3 + C$$

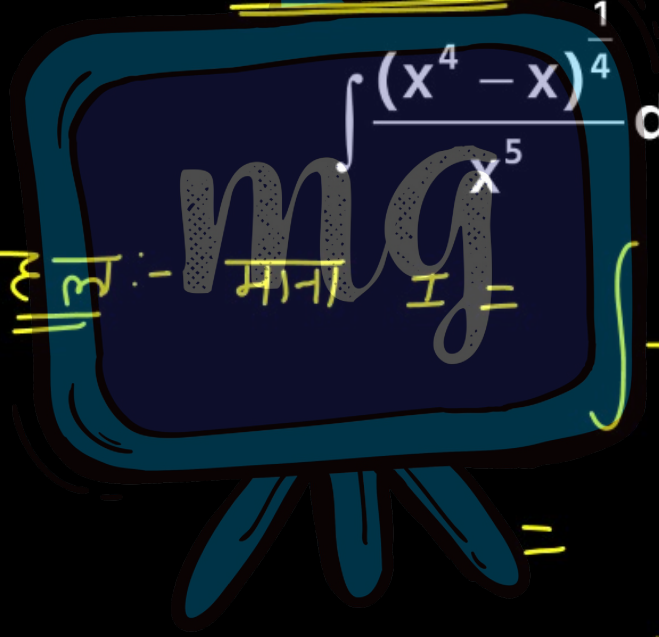
$$= \frac{1}{9}(1+\sin 6x)^{3/2} + C$$



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उदाहरण-36 :

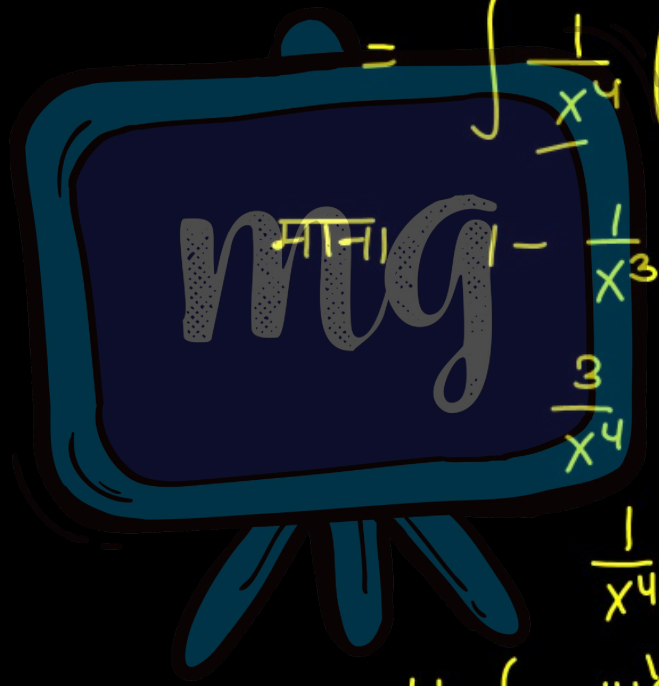
$\int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$ का मान ज्ञात कीजिए।



हल :- माना $I = \int \frac{[x^4 - x]^{\frac{1}{4}}}{x^5} dx$

$= \int \frac{(x^4)^{\frac{1}{4}} \left[1 - \frac{1}{x^3}\right]^{\frac{1}{4}}}{x^5 \, x^4} dx$

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$$= \int \frac{1}{x^4} \left(1 - \frac{1}{x^3}\right)^{1/4} dx$$

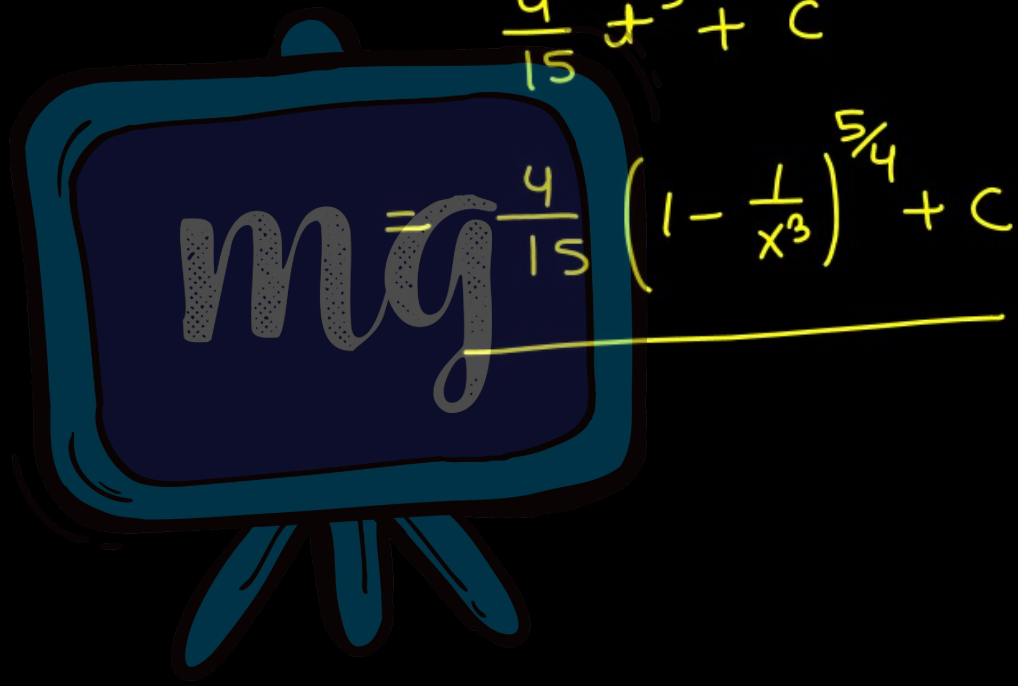
मान लो $1 - \frac{1}{x^3} = t^4$

$$\frac{3}{x^4} = 4t^3 \frac{dt}{dx}$$

$$\frac{1}{x^4} dx = \frac{4}{3} t^3 dt$$

$$= \frac{4}{3} \int (t^4)^{1/4} t^3 dt = \frac{4}{3} \int t^4 dt$$

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उदाहरण-37 :

$\int \frac{x^4 dx}{(x-1)(x^2+1)}$ का मान ज्ञात कीजिए।

हल :- $\frac{x^4}{(x-1)(x^2+1)} = \frac{(x^4-1)+1}{(x-1)(x^2+1)}$

$= \frac{x^4-1}{(x-1)(x^2+1)} + \frac{1}{(x-1)(x^2+1)}$

$= \frac{x+1}{(x-1)(x^2+1)} + \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$

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$$= x+1+\frac{A}{(x-1)}+\frac{Bx+C}{x^2+1}$$

$$\therefore \frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow 1 = A(x^2+1)+(Bx+C)(x-1)$$

$$\Rightarrow 1 = (A+B)x^2+(C-B)x+A-C$$

$$\left.\begin{aligned} A+B &= 0 \\ C-B &= 0 \\ A-C &= 1 \end{aligned}\right\} \Rightarrow A=\frac{1}{2}, \quad B=C=-\frac{1}{2}$$

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$$\frac{x^4}{(x-1)(x^2+1)} = x+1+\frac{1}{2}(x-1) - \frac{1}{2}\left(\frac{x+1}{x^2+1}\right)$$

$$\int \frac{x^4\,dx}{(x-1)(x^2+1)} = \int x\,dx + \int dx + \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{x\,dx}{x^2+1}$$

$$- \frac{1}{2}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1|$$

$$- \frac{1}{2}\tan^{-1}x + C$$