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कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 11

प्रश्नावली 5.5

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उदाहरण-26 : x के सापेक्ष $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ का अवकलन कीजिए।

$$y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$$

दोनो तरफ log लेने पर :

$$\log y = \log \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$$

$$\Rightarrow \log y = \log\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]^{\frac{1}{2}}$$

$$\Rightarrow \log y = \frac{1}{2}\left[\log(x-3) + \log(x^2+4) - \log(3x^2+4x+5)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-3}\frac{d}{dx}(x-3) + \frac{1}{x^2+4}\frac{d}{dx}(x^2+4) - \frac{1}{3x^2+4x+5}\frac{d}{dx}(3x^2+4x+5)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-3} + \frac{1}{x^2+4}(2x) - \frac{1}{3x^2+4x+5}(6x+4)\right]$$

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$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$$

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उदाहरण–27 : x के सापेक्ष a^x का अवकलन कीजिए, जहाँ a एक धन अचर है।

$$y = a^x$$

दोनों तरफ log लेने पर :

$$\log y = \log a^x$$

$$\Rightarrow \log y = x \log a$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times 0 + \log a$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log a$$

$$\Rightarrow \boxed{\frac{dy}{dx} = a^x \log a}$$

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उदाहरण-28 : x के सापेक्ष $x^{\sin x}$, का अवकलन कीजिए, जब कि $x > 0$ है।

$y = x^{\sin x}$

दोनों तरफ log लेने पर :

$\log y = \log x^{\sin x}$

$\Rightarrow \log y = \sin x \cdot \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log x \cdot \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \cos x \cdot \log x$$

$$\Rightarrow \boxed{\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \cdot \log x\right]}$$

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उदाहरण–29 : यदि $y^x + x^y + x^x = a^b$ है। तो $\frac{dy}{dx}$, ज्ञात कीजिए।

$y^x + x^y + x^x = a^b$

$a, b \Rightarrow$ अचर

दोनों तरफ log लेने पर :

$\log y^x + \log x^y + \log x^x = \log a^b$

$\Rightarrow x \log y + y \log x + x \log x = b \log a$

$$\Rightarrow x\left[\frac{1}{y}\frac{dy}{dx}\right] + \log y \cdot (1) + y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} + \cancel{x} \cdot \cancel{\frac{1}{x}} + \log x = 0$$

$$\Rightarrow \frac{x}{y}\frac{dy}{dx} + \log x \frac{dy}{dx} = -\frac{y}{x} - 1 - \log x - \log y$$

$$\Rightarrow \left(\frac{x}{y} + \log x\right)\frac{dy}{dx} = -\left(\frac{y}{x} + 1 + \log x + \log y\right)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-\left(\frac{y}{x} + 1 + \log x + \log y\right)}{\frac{x}{y} + \log x}}$$

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प्रश्नावली 5.5

1 से 11 तक के प्रश्नों में प्रदत्त फलनों का x के सापेक्ष अवकलन कीजिए:

1. cos x . cos 2x . cos 3x

$y = \cos x \cdot \cos 2x \cdot \cos 3x$

दोनों तरफ log लेने पर:

$\log y = \log(\cos x \cdot \cos 2x \cdot \cos 3x)$

$\Rightarrow \log y = \log \cos x + \log \cos 2x + \log \cos 3x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\frac{d}{dx}(\cos x) + \frac{1}{\cos 2x}\frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x}\frac{d}{dx}(\cos 3x)$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}$$

$$\Rightarrow \frac{dy}{dx} = \cos x \cdot \cos 2x \cdot \cos 3x \left[-\frac{\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x} \right]$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\cos x \cdot \cos 2x \cdot \cos 3x \left[\frac{\sin x}{\cos x} + \frac{2\sin 2x}{\cos 2x} + \frac{3\sin 3x}{\cos 3x} \right]}$$

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$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

2. $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

दोनों तरफ log लेने पर:

$$\log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

