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Class - 10

Mathematics

Chapter - 2

Polynomial

Part - 2

Harsh Goyal

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$p(x) = x^2 + 1$

$x = 2$

$p(2) = (2)^2 + 1$

$= 4 + 1$

$= 5$

$p(2) = 5$

VALUE OF A POLYNOMIAL

If p(x) is a polynomial in x, and if k is any any real real number, n then the value obtained by replacing x by k in p(x) is Called the value of p(x) at x = k and is denoted by p(k)

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Values of polynomial $p(x) = x^2 - 3x - 4$ at $x = 1$ and $x = 4$ are :

$p(x) = x^2 - 3x - 4$

$p(1) = (1)^2 - 3(1) - 4$

$= 1 - 3 - 4$

$= -6$

$x = 4$

$(4)^2 - 3(4) - 4$

$= 16 - 12 - 4$

$= 0$

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ZERO OF A POLYNOMIAL

- A real number k is said to be a zero of real number a polynomial p(x), if p(k) = 0.
- Finding a zero of a polynomial p(x)

a polynomial p(x)

$x = k$ $p(x)$

$p(k) = 0$

$(-1, 1)$

$p(x) = x^2 - 1$ $p(x)$

$1 - 1 = 0$

$1 - 1 = 0$

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EXAMPLE :

For the values of x given in front of the following polynomials, check which of the polynomials has zeroes.

(i) $4x + 2$ $\quad x = -\frac{1}{2}$

$4\left(-\frac{1}{2}\right) + 2$

$-2 + 2$

$= 0$

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(ii) $2y^2 - 3y + 4$ $\boxed{y = 2}$

$2(2)^2 - 3(2) + 4$

$= 2 \times 4 - 6 + 4$

$= 8 - 6 + 4$

$= 12 - 6$

$P(2) = 6 \neq 0$

$y = 2$ is not a zero

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(iii) $5x^3 + 4x^2 + x - \sqrt{2}$ $\quad x = 3$

$5(3)^3 + 4(3)^2 + 3 - \sqrt{2}$

$= 5 \times 27 + 4 \times 9 + 3 - \sqrt{2}$

$\neq 0$

$x = 3$ is not a zero of this poly.

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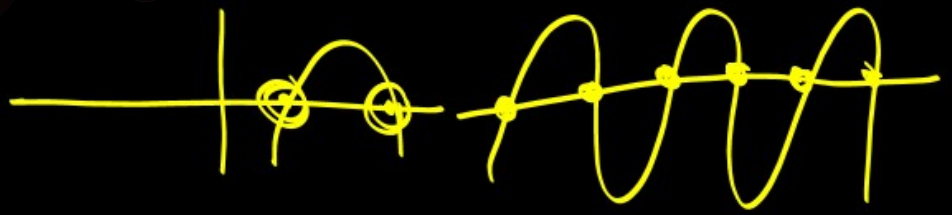
GEOMETRICAL MEANING OF THE ZEROES OF A POLYNOMIAL

$y = x^2$

$p(x) = x^2$

$x = 0$

❖ Let y = f(x) be a polynomial, then the points at which the graph of y = f(x) intersects the x-axis are called zeroes of the polynomial.

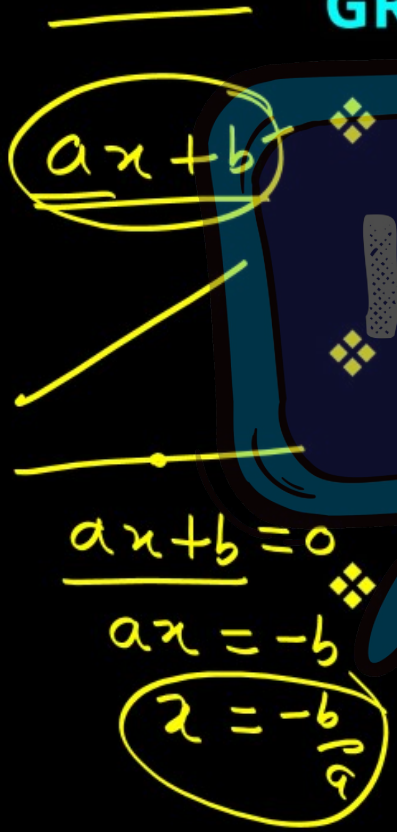


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GRAPH OF A LINEAR POLYNOMIAL



- In general, for a linear polynomial $ax + b$, $a \neq 0$
- the graph of $y = ax + b$ is a straight line.
- it intersects the x-axis at exactly one point, namely $\left(\frac{-b}{a}, 0\right)$.

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EXAMPLE :

Make a graphical representation of the following linear polynomial.

$f(x) = 2x + 3$

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

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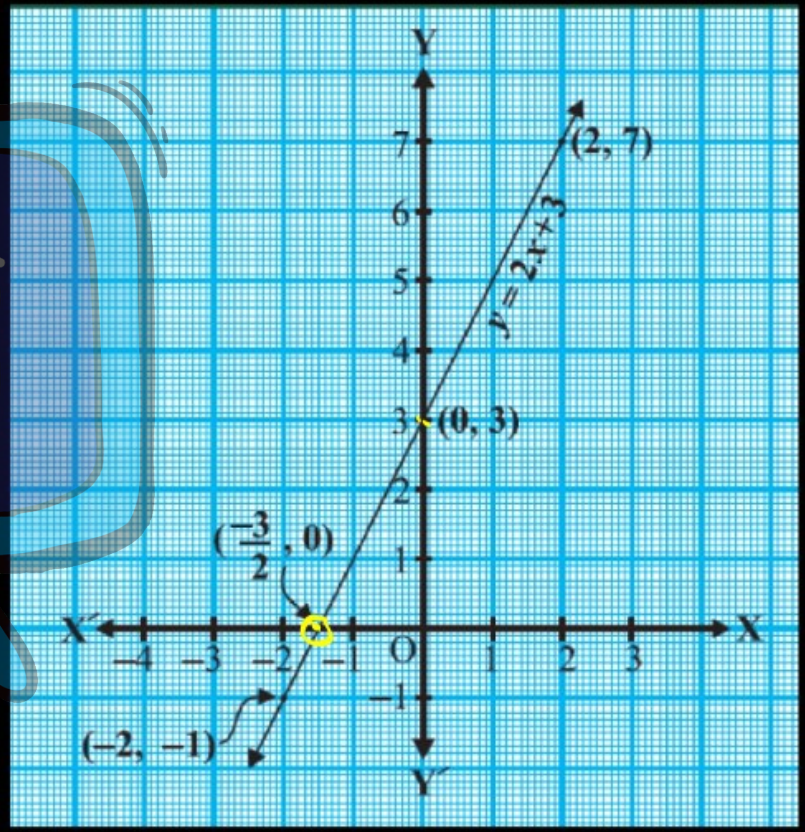
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$y = 2x + 3$

x	0	$-\frac{3}{2}$
y	3	0

$(0, 3)$

$x = -\frac{3}{2}$



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GRAPH OF A QUADRATIC POLYNOMIAL

For any quadratic polynomial

$ax^2 + bx + c$, $a \neq 0$

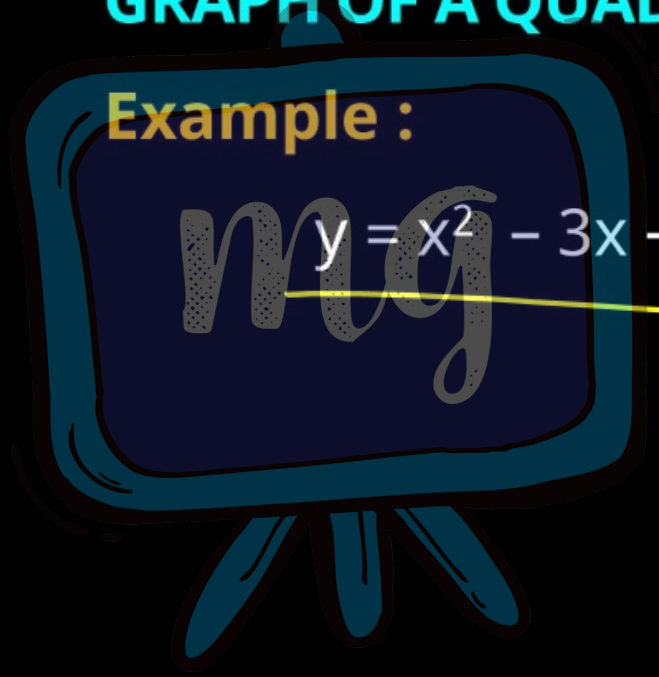
- the graph is a curve called parabola.
- it opens upwards like  if $a > 0$.
- it opens downwards like  if $a > 0$.

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GRAPH OF A QUADRATIC POLONOMIAL



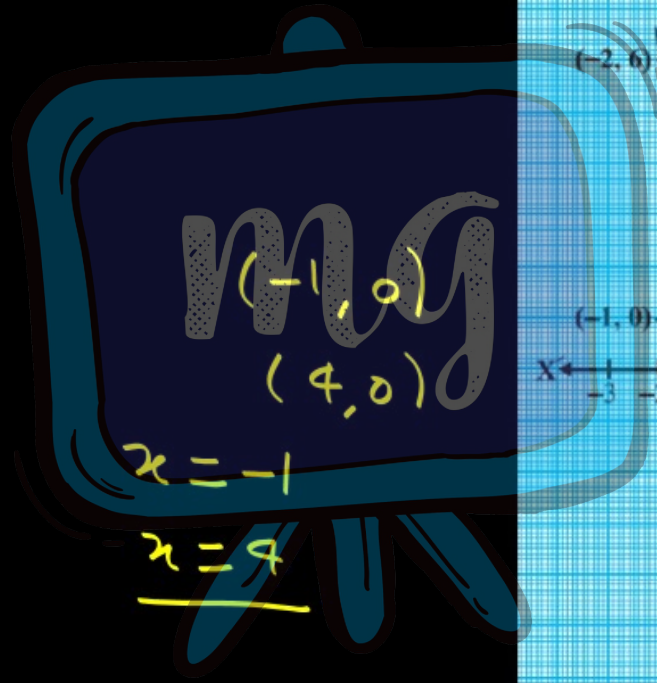
Example :

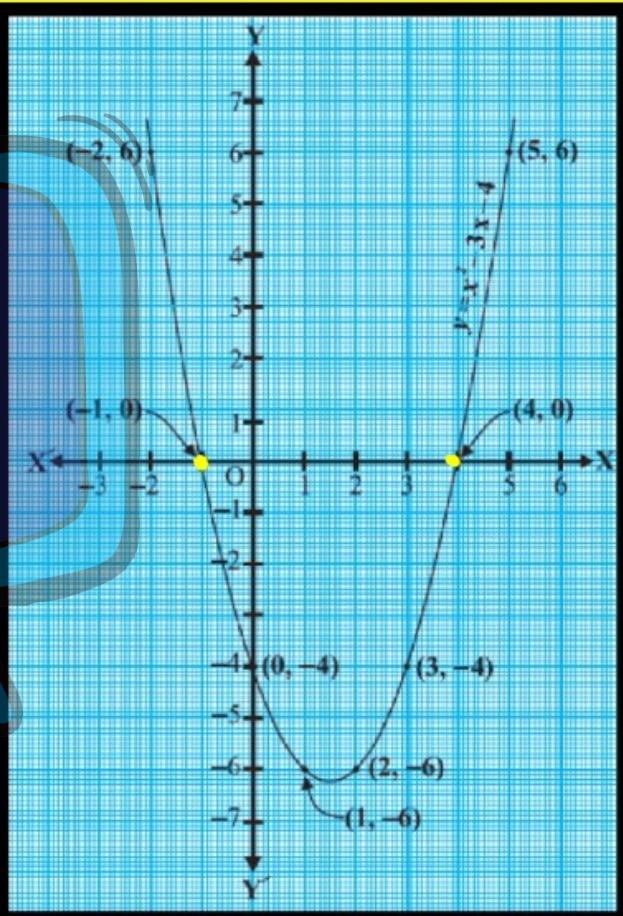
$y = x^2 - 3x - 4$

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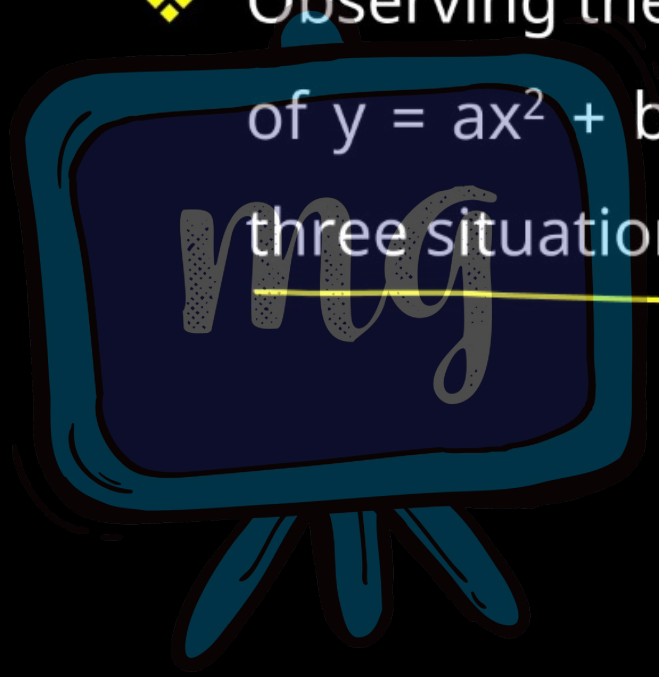


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- Observing the shape of the graph of $y = ax^2 + bx + c$, the following three situations are possible.

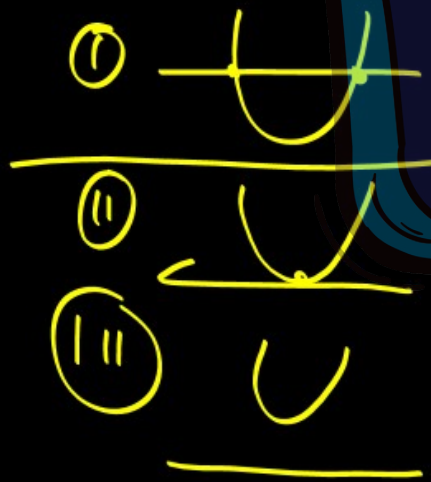


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Case (i) : The graph cuts x – axis at two distinct points.

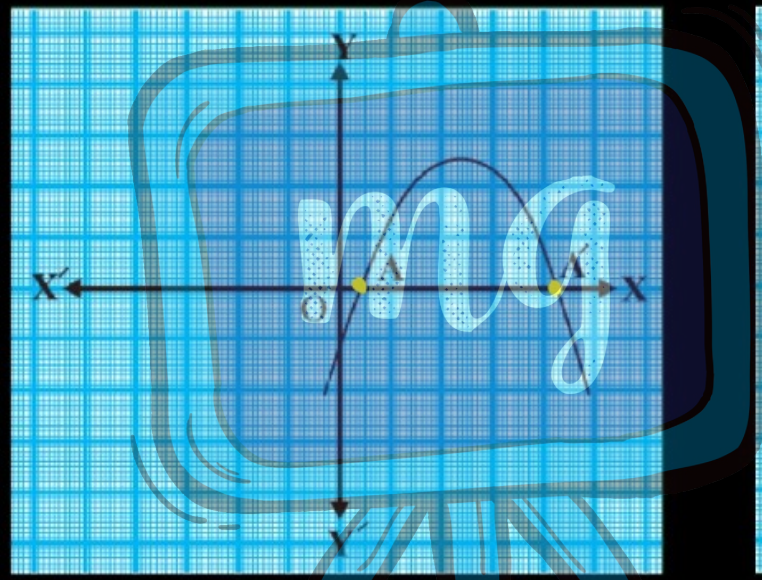


- The graph cuts x – axis at two distinct points A and A'.
- the x-coordinates of these points are the two zeroes of the polynomial $ax^2 + bx + c$.

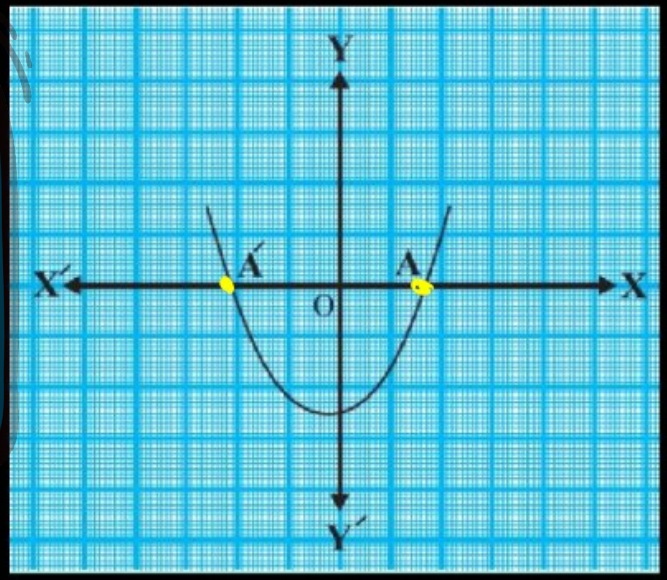
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(i)



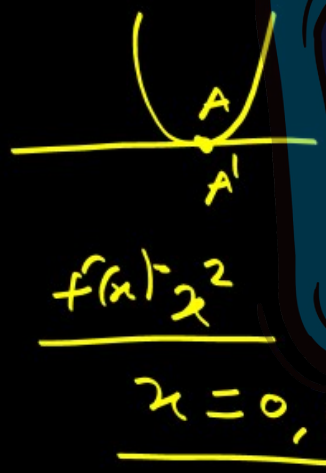
(ii)

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Case (ii) : The graph cuts x axis at exactly one-point.

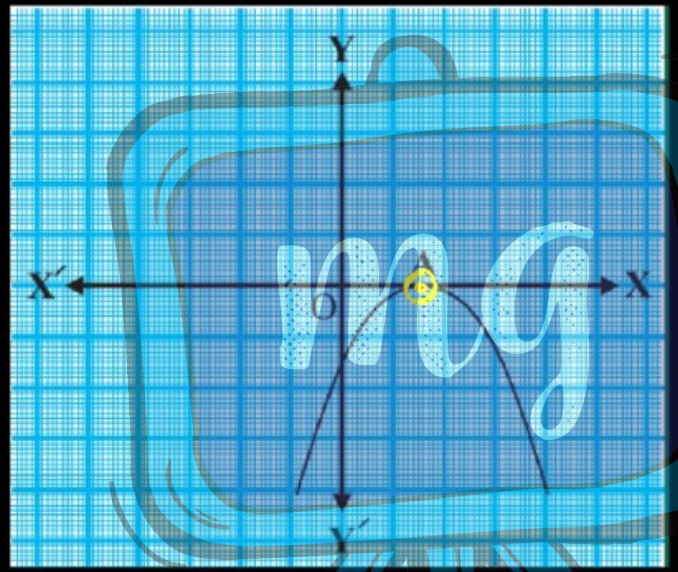


- The two points A and A' coincide.
- the x – coordinate of this point gives two equal zeroes of the polynomial $ax^2 + bx + c$.

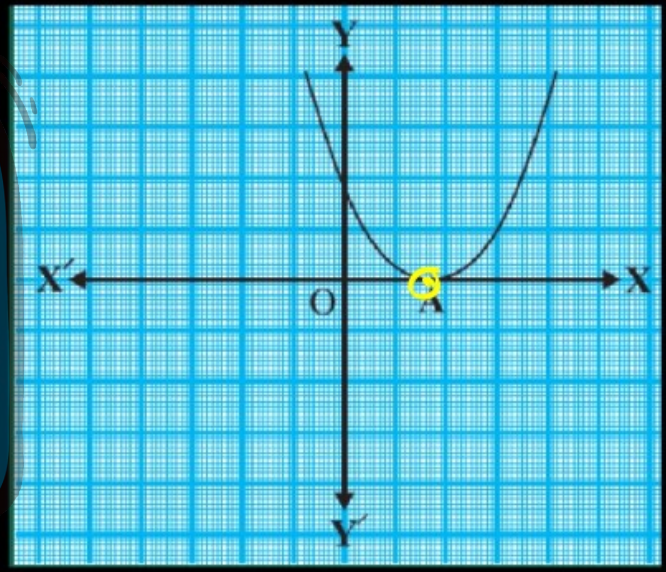
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(i)



(ii)

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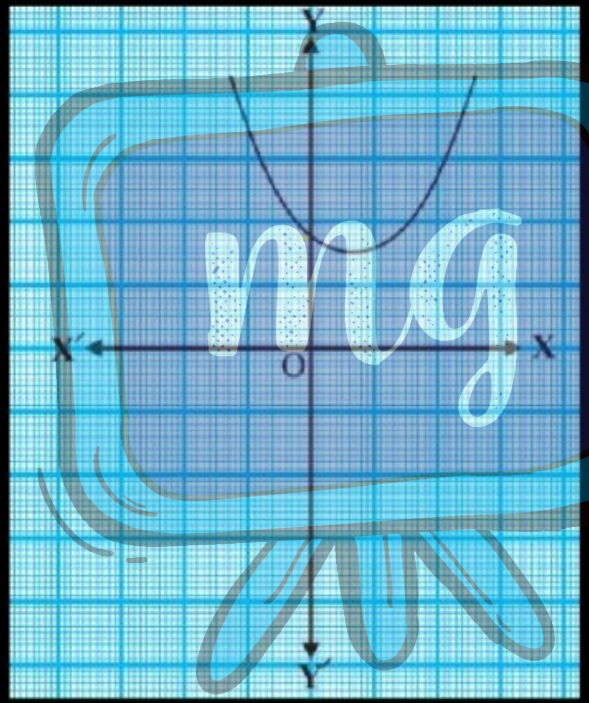
Case (iii) : The graph does not cut the x - axis.

- The parabola $y = ax^2 + bx + c$ opens upwards and remains completely above x – axis, if $a > 0$
- The parabola $y = ax^2 + bx + c$ opens downwards and remains completely below x – axis, if $a < 0$.

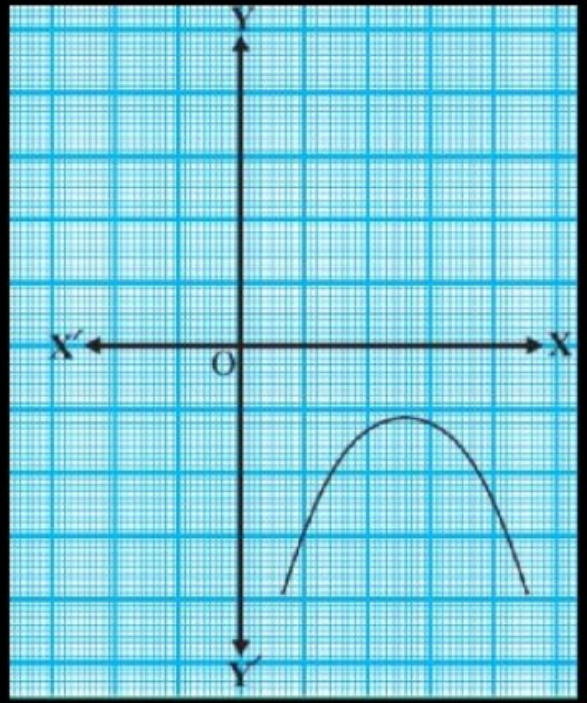
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(i)



(ii)

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CONCLUSION

A polynomial of degree 2 has atmost two zeroes.

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GRAPH OF A CUBIC POLYNOMIAL

- The general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$, $a \neq 0$.
- A polynomial of degree 3 can have atmost three zeroes.

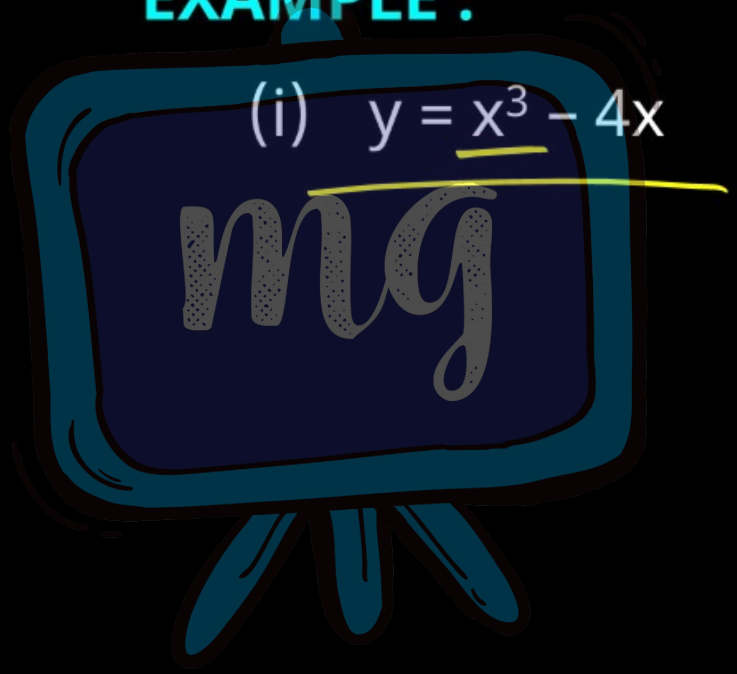
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EXAMPLE :

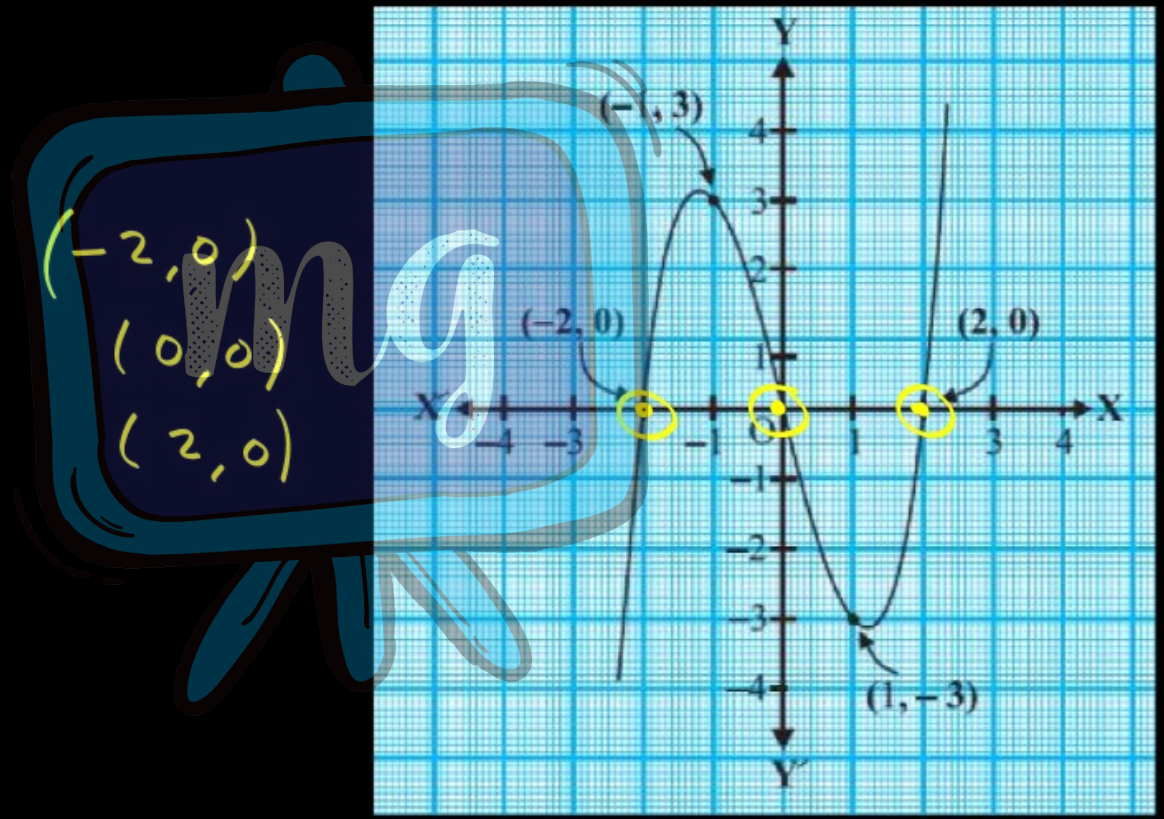
(i) $y = x^3 - 4x$



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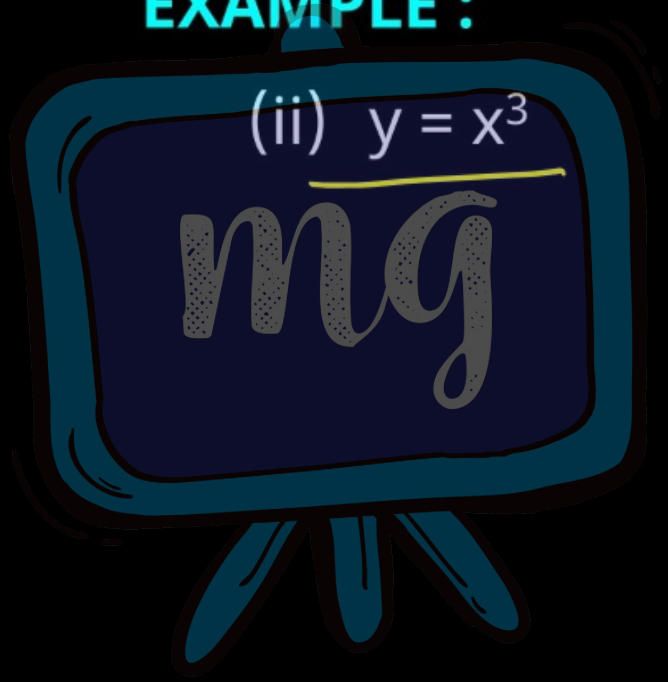


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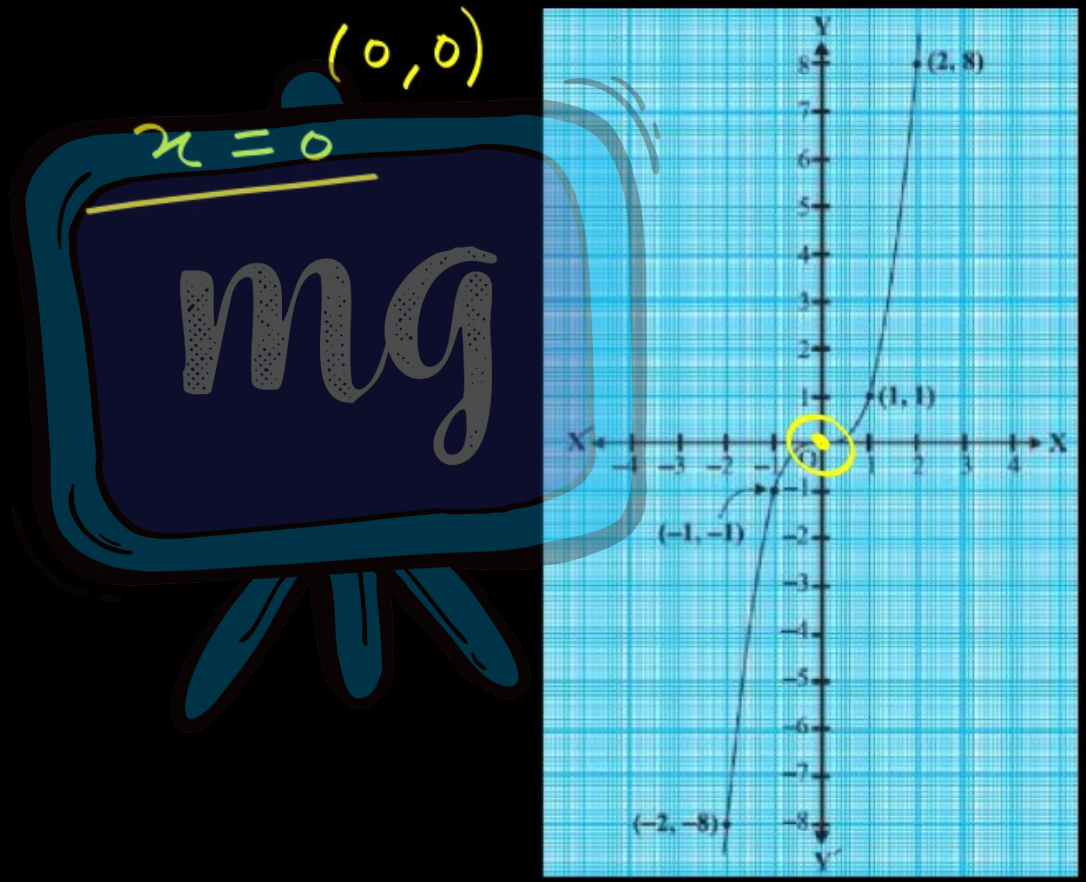
EXAMPLE :



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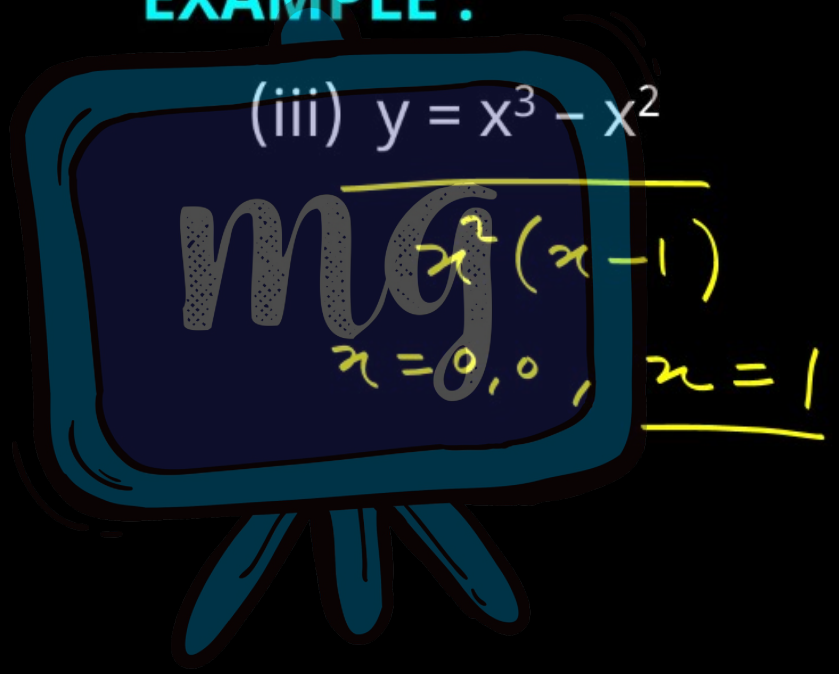


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EXAMPLE :

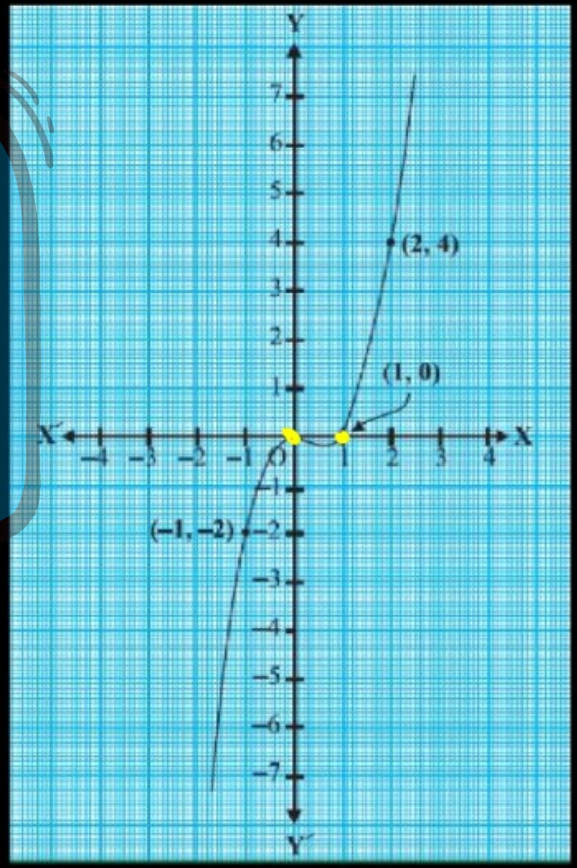


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CONCLUSION

- The graph of a linear polynomial crosses x – axis at one point only and the polynomial has one zero.

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- The graph of a quadratic polynomial crosses x axis at atmost two points, and the polynomial has atmost two zeroes.

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- The graph of a cubic polynomial crosses X- axis at atmost three points, and the polynomial has atmost three zeroes.

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- The graph of a polynomial of degree n crosses x – axis at atmost n points, and the polynomial has atmost n zeroes.

$p(x) = x^6 + x^5 + 2x + 1$

atmost

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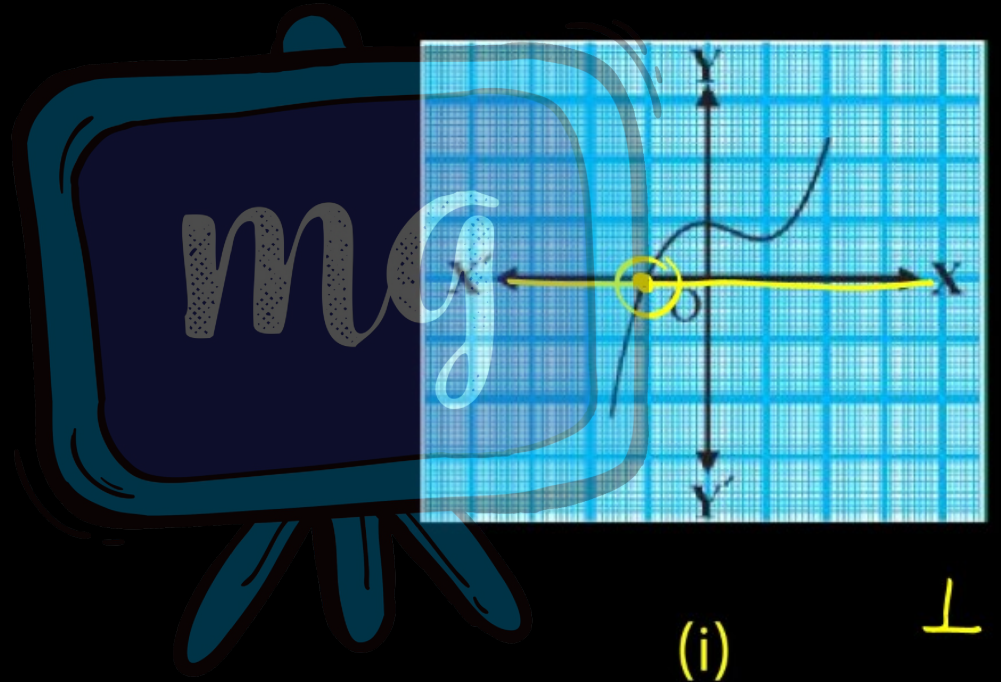
EXAMPLE 1 :

Look at the graphs given. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).

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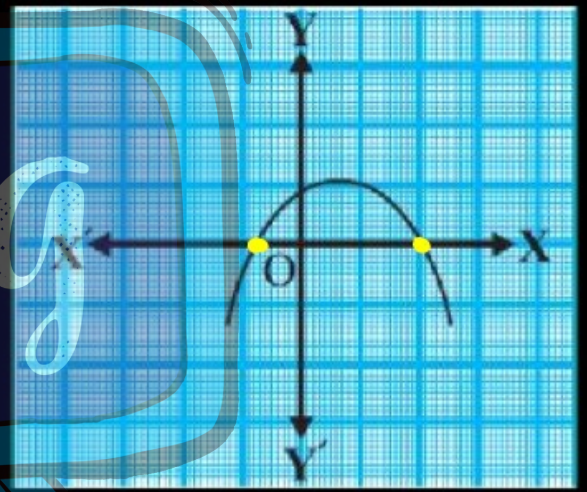


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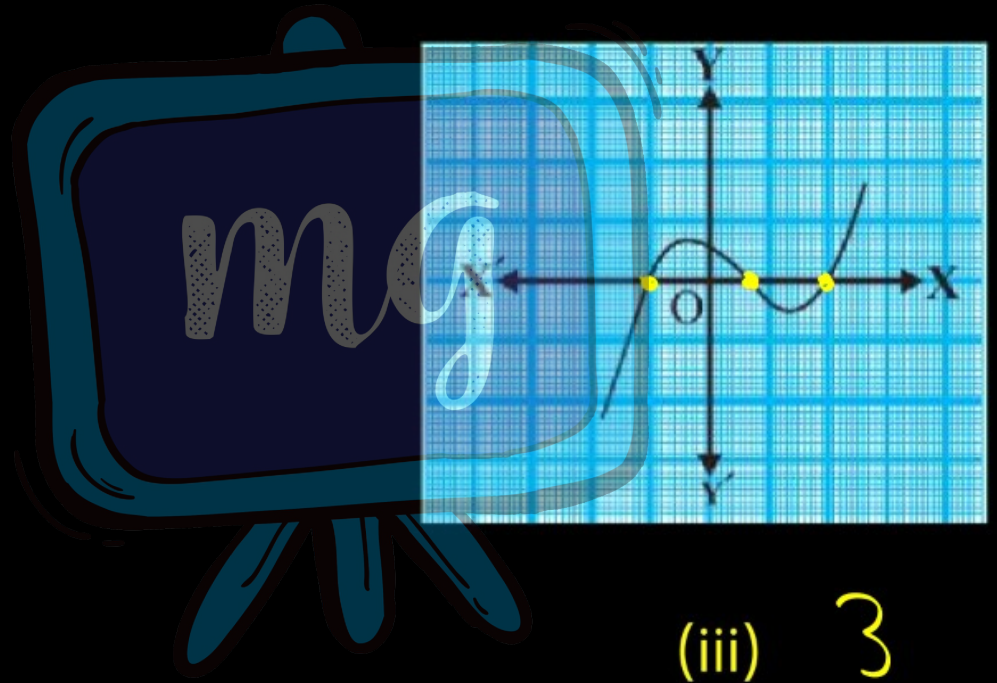


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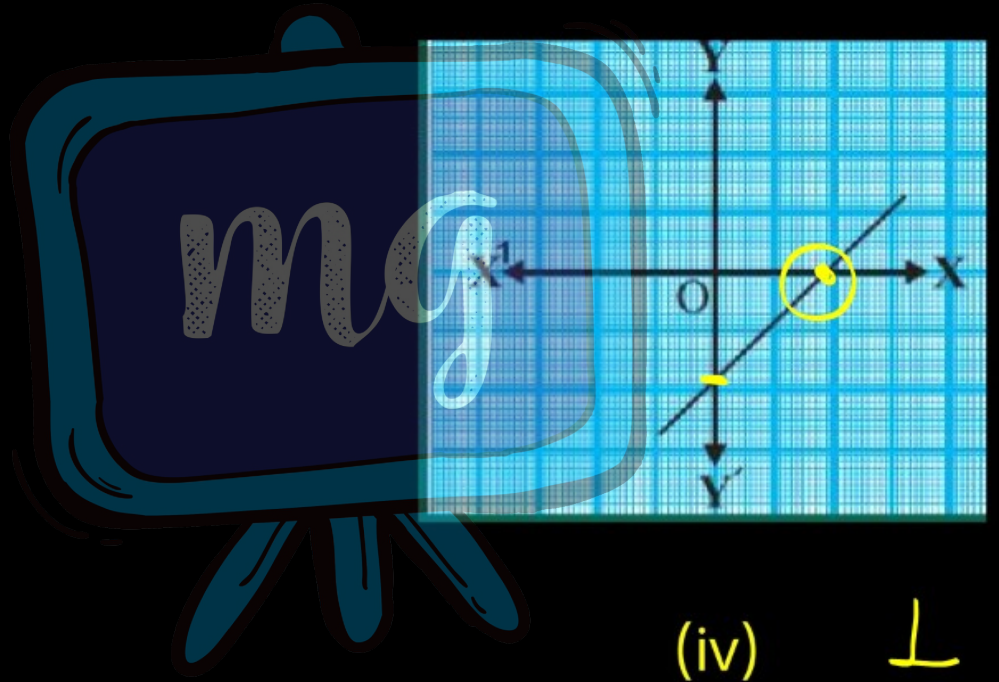


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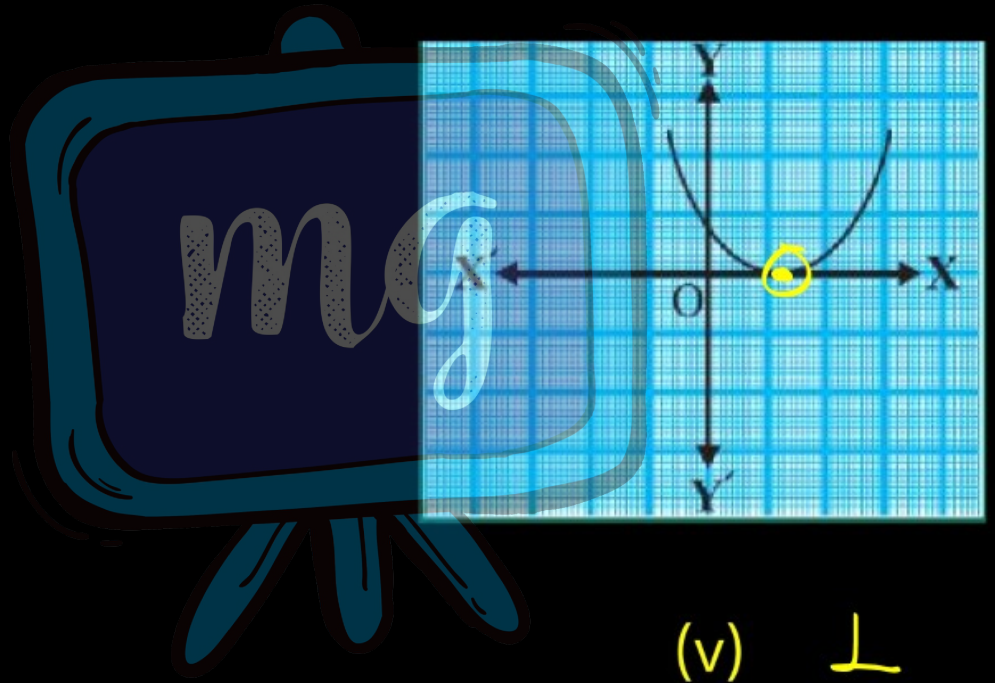


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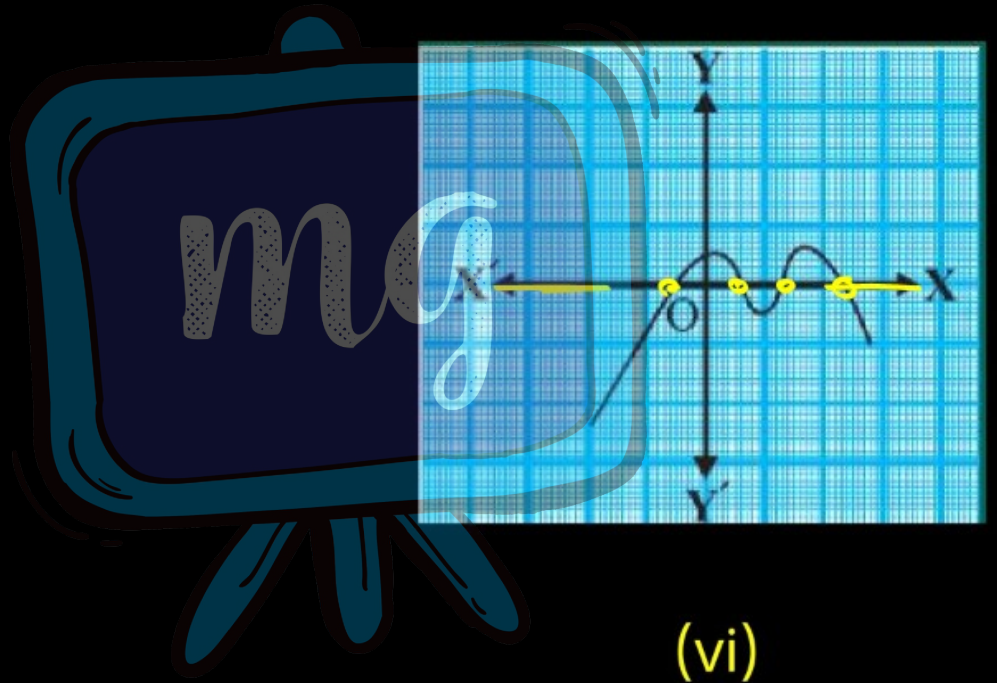


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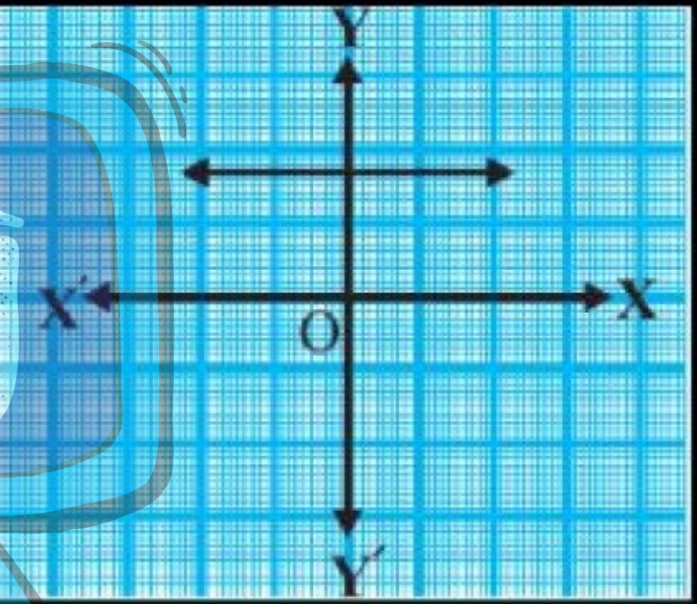
EXERCISE 2.1

1. The graphs of $y = p(x)$ are given, for some polynomials $p(x)$. Find the number of zeroes of p(x), in each case.

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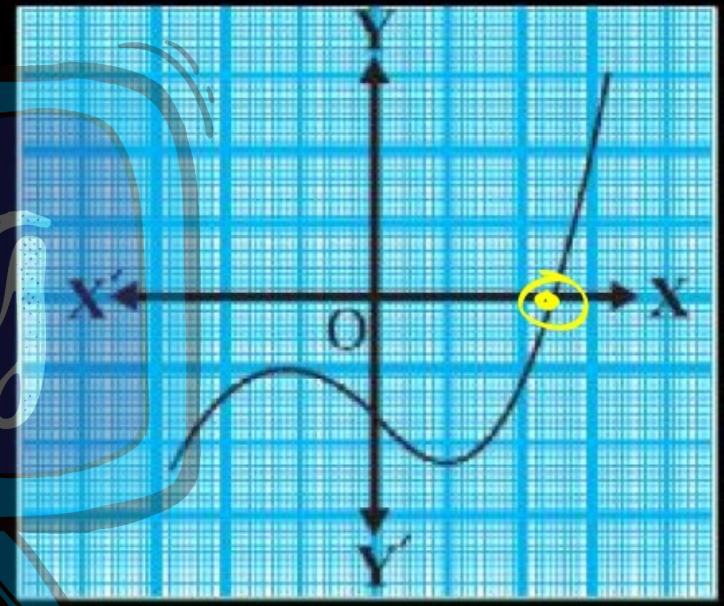


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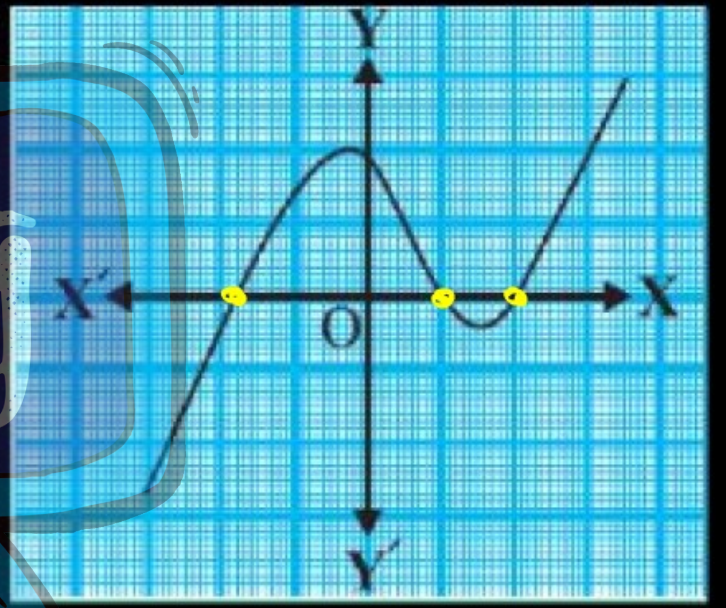


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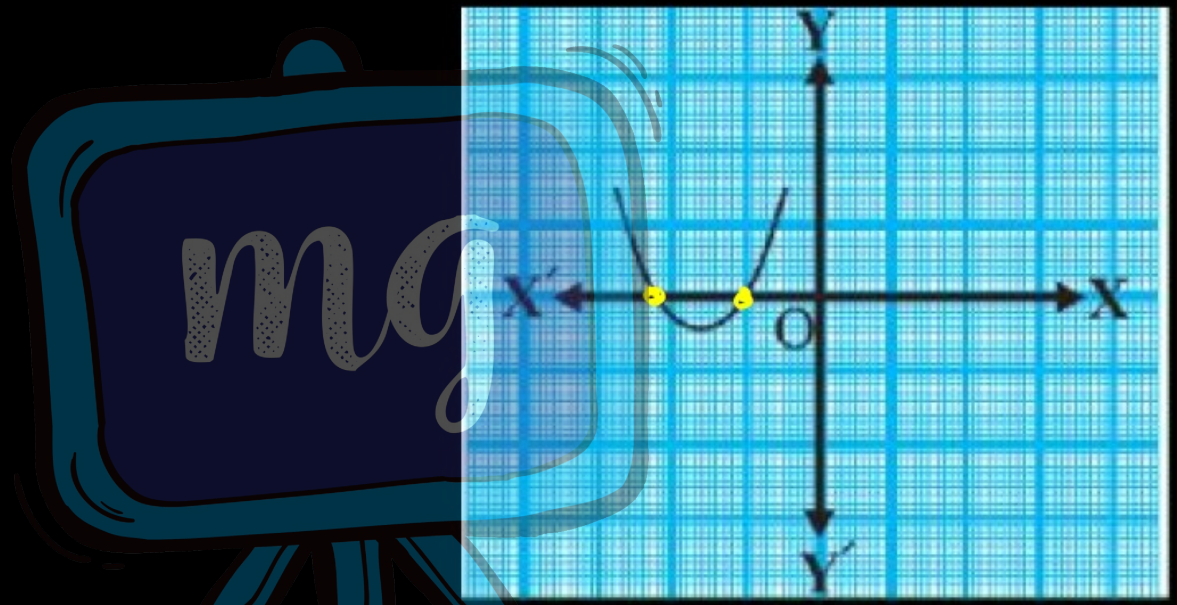


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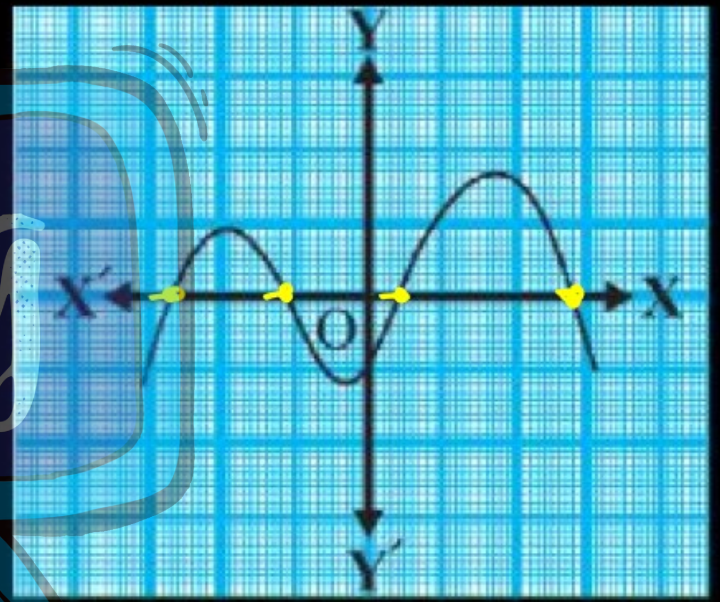


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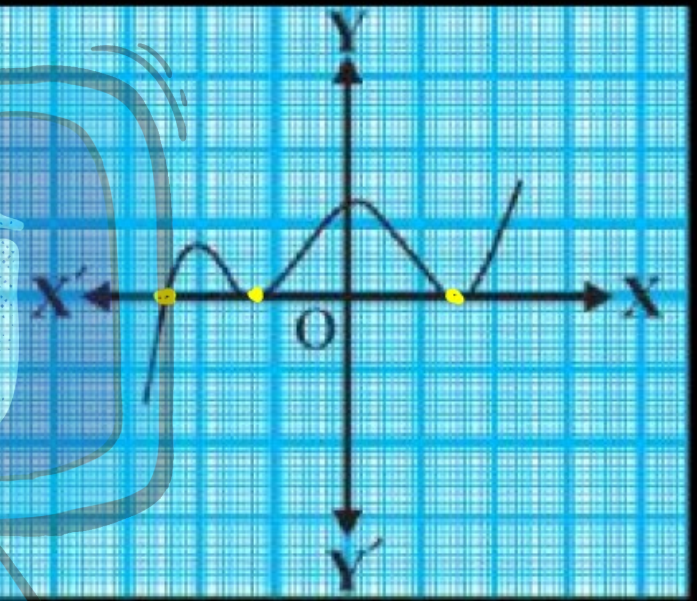


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(vi) 3

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LINEAR POLYNOMIAL

- Let $f(x) = ax + b (a \neq 0)$ be a linear polynomial and it has a zero, then

$ax + b = 0$

$x = -b/a$

$$\alpha = \frac{-b}{a}$$

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OR

$$\text{ZERO OF LINEAR POLYNOMIAL} = \frac{-\text{constant term}}{\text{coefficient of x}}$$

$2x + 5$

$\frac{-5}{2}$

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RELATIONSHIP BETWEEN ZEROES AND COEFFICIENTS OF QUADRATIC POLYNOMIAL:-

$ax^2 + bx + c$

$\alpha + \beta = \frac{-b}{a}$

$\alpha \cdot \beta = \frac{c}{a}$



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$p(x) = 2x^2 + 6x + 12$

$\alpha \quad \beta$

$\alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = \frac{-6}{2}$

$\alpha + \beta = -3$

$a = 2, \; b = 6, \; c = 12$

$\alpha \cdot \beta = \frac{c}{a} = \frac{12}{2}$

$\alpha\beta = 6$