कक्षा – 12

गणित

समाकलन

अध्याय – 7

भाग - 8

प्रश्नावली 7.4

By : Rajendra Yadav Sir

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प्रश्नावली 7.4

$$\int \frac{3x^2}{x^6+1}\,dx$$

$$= \int \frac{3x^2}{(x^3)^2+1}\,dx$$

$$= \int \frac{dt}{t^2+(1)^2}$$

➤ प्रश्न 1 से 23 तक के फलनों का समाकलन कीजिए।

Q1. $\dfrac{3x^2}{x^6+1}$

माना $x^3 = t$

$\Rightarrow 3x^2\,dx = dt$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

$$= \frac{1}{1} \tan^{-1}\left(\frac{t}{1}\right) + c$$

$$= \tan^{-1} t + c$$

$$= \tan^{-1}(x^3) + c$$



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$$\int \frac{1}{\sqrt{1+4x^2}}\,dx$$

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$$= \int \frac{1}{\sqrt{1+(2x)^2}}\,dx$$

$$= \frac{1}{2}\log\left|2x+\sqrt{(2x)^2+1}\right|+C$$

Q2. $\int \frac{1}{\sqrt{1+4x^2}}\,dx$

$$= \int \frac{1}{\sqrt{x^2+a^2}}\,dx$$

$$= \log\left|x+\sqrt{x^2+a^2}\right|+C$$

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$$\int \frac{1}{\sqrt{(2-x)^2+1}}\,dx$$

Q3. $\dfrac{1}{\sqrt{(2-x)^2+1}}$

माना $2-x = t$

$\Rightarrow -dx = dt$

$\Rightarrow dx = -dt$

$$= -\int \frac{dt}{\sqrt{t^2+1}}$$

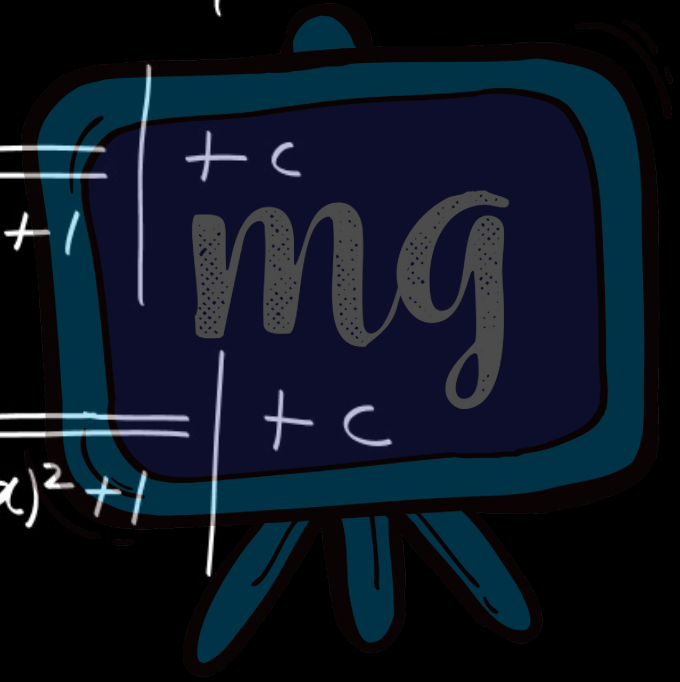
$$= -\log\left|t+\sqrt{t^2+1}\right| + c$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$= -\log\left|t+\sqrt{t^2+1}\right| + c$$

$$= \log\left|\frac{1}{t+\sqrt{t^2+1}}\right| + c$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{(2-x)^2+1}}\right| + c$$



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Q4. $\frac{1}{\sqrt{9-25x^2}}$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$$

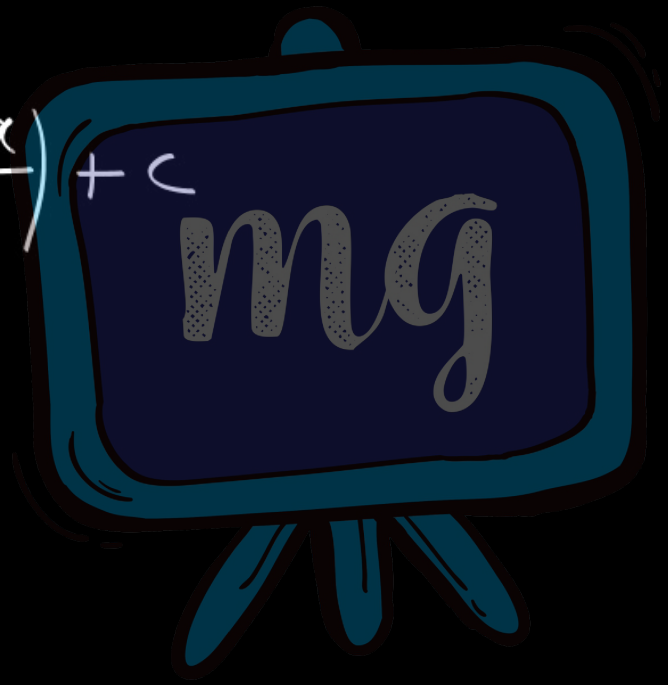
$$\int \frac{1}{\sqrt{9-25x^2}}\,dx$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{\frac{9}{25}-x^2}}\,dx$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2-x^2}}\,dx$$

$$= \frac{1}{5} \sin^{-1}\left(\frac{x}{3/5}\right) + c$$

$$= \frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + c$$



प्रश्नावली 7.4

$$\int \frac{3x}{\sqrt{1+2x^4}}\,dx$$

➤ प्रश्न 1 से 23 तक के फलनों का समाकलन कीजिए।

Q5. $\dfrac{3x}{\sqrt{1+2x^4}}$

मान $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \dfrac{dt}{2}$

$\Rightarrow 3x\,dx = \dfrac{3}{2}dt$

$$\int \frac{3x}{\sqrt{1+2(x^2)^2}}\,dx$$

$$= \frac{3}{2}\int \frac{dt}{\sqrt{1+2t^2}}$$

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$$= \frac{3}{2} \int \frac{dt}{\sqrt{1+2t^2}}$$

$$= \frac{3}{2\sqrt{2}} \int \frac{dt}{\sqrt{\frac{1}{2}+t^2}}$$

$$= \frac{3}{2\sqrt{2}} \int \frac{dt}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+t^2}}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$= \frac{3}{2\sqrt{2}} \log\left|t+\sqrt{t^2+\left(\frac{1}{\sqrt{2}}\right)^2}\right| + c$$

$$= \frac{3}{2\sqrt{2}} \log\left|x^2+\sqrt{x^2+\frac{1}{2}}\right| + c$$

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$$\int \frac{x^2}{1-x^6}\,dx$$

$$= \int \frac{x^2}{1-(x^3)^2}\,dx$$

$$= \frac{1}{3}\int \frac{dt}{(1)^2 - t^2}$$

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Q6. $\dfrac{x^2}{1-x^6}$

माना $x^3 = t$

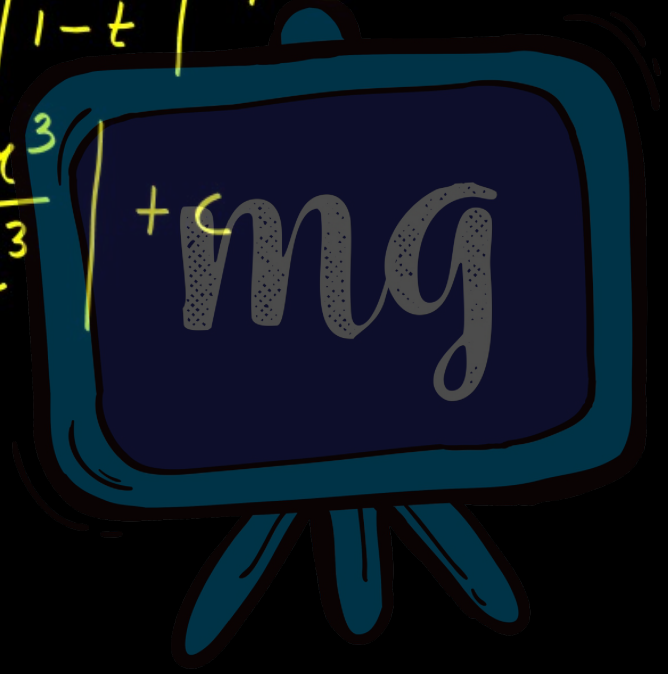
$3x^2\,dx = dt$

$\Rightarrow x^2\,dx = \dfrac{dt}{3}$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

$$= \frac{1}{3}\left(\frac{1}{2\times 1}\right)\log\left|\frac{1+t}{1-t}\right| + c$$

$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + c$$



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Q7. $\dfrac{x-1}{\sqrt{x^2-1}}$

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx$$

$$= \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{dx}{\sqrt{x^2-1}}$$

$$= I_1 - I_2$$

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अब, $I_1 = \int \frac{x}{\sqrt{x^2-1}}\,dx$

माना $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{\sqrt{t^2-1}}$

$= \frac{1}{2}\log\left|t+\sqrt{t^2-1}\right| + C$

$I_1 = \frac{1}{2}\log\left|x^2+\sqrt{x^2-1}\right| + C$

$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

इसीप्रकार, $I_2 = \int \frac{dx}{\sqrt{x^2-1}}$

$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c$

$I_2 = \log\left|x+\sqrt{x^2-1}\right| + c$

अतः $I = I_1 - I_2$

$I = \frac{1}{2}\log\left|x^2+\sqrt{x^2-1}\right| - \log\left|x+\sqrt{x^2-1}\right| + c$

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$$\int \frac{x^2}{\sqrt{x^6+a^6}}\,dx$$

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Q8. $\int \frac{x^2}{\sqrt{x^6+a^6}}\,dx$

$$= \int \frac{x^2}{\sqrt{(x^3)^2+(a^3)^2}}\,dx$$

माना $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\Rightarrow x^2dx = \frac{1}{3}dt$

$$= \frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|+c$$

$$= \frac{1}{3} \log \left| t + \sqrt{t^2 + (a^3)^2} \right| + c$$

$$= \frac{1}{3} \log \left| (x^3) + \sqrt{(x^3)^2 + (a^3)^2} \right| + c$$

$$= \frac{1}{3} \log \left| x^3 + \sqrt{x^6 + a^6} \right| + c$$

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$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$$

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Q9. $\dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

माना $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$$\int \frac{dt}{\sqrt{t^2+4}}$$

$$= \int \frac{dt}{\sqrt{t^2 + (2)^2}}$$

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$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \frac{dt}{\sqrt{t^2+(2)^2}} = \log\left|t+\sqrt{t^2+(2)^2}\right| + C$$

$$= \log\left|t+\sqrt{t^2+4}\right| + C$$

$$= \log\left|\tan x+\sqrt{\tan^2 x+4}\right| + C$$

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Q10. $\int \frac{1}{\sqrt{x^2+2x+x}}\,dx$

$= \int \frac{1}{\sqrt{x^2+3x}}\,dx$

$= \int \frac{1}{\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}}\,dx$

माना $x+\frac{3}{2}=t$

$\Rightarrow dx = dt$

x^2+2x+x

$x^2+3x+\frac{9}{4}-\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|+c$$

$$\int \frac{dt}{\sqrt{t^2-\left(\frac{3}{2}\right)^2}}$$

$$= \log\left|t+\sqrt{t^2-\left(\frac{3}{2}\right)^2}\right|+c$$

$$= \log\left|\left(x+\frac{3}{2}\right)+\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}\right|+c$$

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➤ प्रश्न 1 से 23 तक के फलनों का समाकलन कीजिए।

$$\int \frac{1}{9x^2+6x+5}\,dx$$

$$= \int \frac{1}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}\,dx$$

$$= \int \frac{dt}{t^2+\left(\frac{2}{3}\right)^2}$$

Q11. $\dfrac{1}{9x^2+6x+5}$

मान $x+\frac{1}{3}=t$

$dx=dt$

$9x^2+6x+5$

$x^2+\frac{6}{9}x+\frac{5}{9}$

$x^2+\frac{6}{9}x+\frac{1}{9}-\frac{1}{9}+\frac{5}{9}$

$\left(x+\frac{1}{3}\right)^2+\frac{4}{9}$

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$$\int \frac{dt}{t^2+\left(\frac{2}{3}\right)^2}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

$$= \frac{1}{2/3}\tan^{-1}\left(\frac{t}{2/3}\right)+c \Rightarrow \frac{3}{2}\tan^{-1}\left[\frac{3}{2}\left(x+\frac{1}{3}\right)\right]+c$$

$$= \frac{3}{2}\tan^{-1}\left(\frac{3t}{2}\right)+c$$