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कक्षा – 12

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प्रश्नावली 7.3

By : Rajendra Yadav Sir

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उदाहरण – 7

➤ निम्नलिखित को ज्ञात कीजिए

(i) $\int \cos^2 x \, dx$

$= \int \frac{\cos 2x + 1}{2} dx$

$= \frac{1}{2}\int \cos 2x \, dx + \frac{1}{2}\int 1 \cdot dx$

$= \frac{1}{2} \frac{\sin 2x}{2} + \frac{x}{2} + C$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = \cos 2\theta + 1$

$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$

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$$= \frac{1}{4}\sin 2x + \frac{x}{2} + c$$



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➤ निम्नलिखित को ज्ञात कीजिए

(ii) $\int \sin 2x \cos 3x \, dx$

$\left[\begin{array}{l} 2 \sin A \cos B \\ = \sin(A+B) + \sin(A-B) \end{array}\right.$

$= \frac{1}{2}\int 2 \sin 2x \cdot \cos 3x \, dx$

$= \frac{1}{2}\int [\sin 5x + \sin(2x-3x)] \, dx$

$= \frac{1}{2}\int [\sin 5x - \sin x] \, dx$

$$= \frac{1}{2}\int \sin 5x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(-\frac{\cos 5x}{5}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \boxed{-\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C}$$

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उदाहरण – 7

➤ निम्नलिखित को ज्ञात कीजिए

(iii) $\int \sin^3 x \, dx$

$$\int \frac{3\sin x - \sin 3x}{4} dx$$

$$= \frac{3}{4}\int \sin x \, dx - \frac{1}{4}\int \sin 3x \, dx$$

$$= -\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$4\sin^3\theta = 3\sin\theta - \sin 3\theta$

$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$

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M2

$$\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx$$

$$= \int (1-\cos^2 x)\cdot \sin x\,dx$$

$$= -\int (1-t^2)\,dt$$

$$= \int (t^2-1)\,dt$$

$$= \int t^2\,dt - \int 1\cdot dt$$

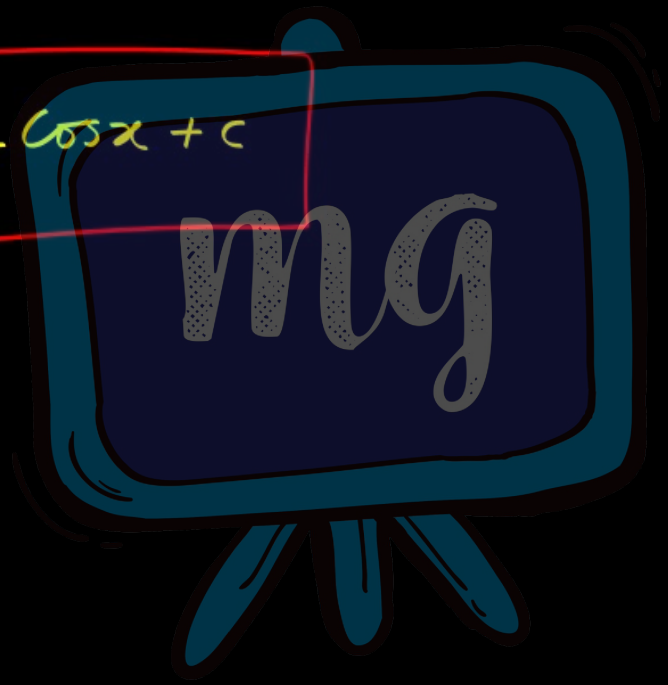
माना $\cos x = t$

$\Rightarrow -\sin x\,dx = dt$

$\Rightarrow \sin x\,dx = -dt$

$$= \frac{t^3}{3} - t + c$$

$$= \frac{\cos^3 x}{3} - \cos x + c$$



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प्रश्नावली 7.3

$\int \sin^2(2x+5)\,dx$ ➤

1 से 22 तक के प्रश्नों में प्रत्येक फलन का समाकलन ज्ञात कीजिए।

Q1. $\sin^2(2x+5)$

$$= \int \frac{1-\cos 2(2x+5)}{2}\,dx$$

$$= \int \frac{1-\cos(4x+10)}{2}\,dx$$

$$= \frac{1}{2}\int 1\cdot dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$\cos 2\theta = 1-2\sin^2\theta$

$2\sin^2\theta = 1-\cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1-2\sin^2\theta$

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प्रश्नावली 7.3

$\int \cos 4x \cdot \sin 3x \, dx$

1 से 22 तक के प्रश्नों में प्रत्येक फलन का समाकलन ज्ञात कीजिए।

$= \frac{1}{2}\int 2 \cos 4x \sin 3x \, dx$

Q2. $\sin 3x \cos 4x \Leftrightarrow \cos 4x \cdot \sin 3x$

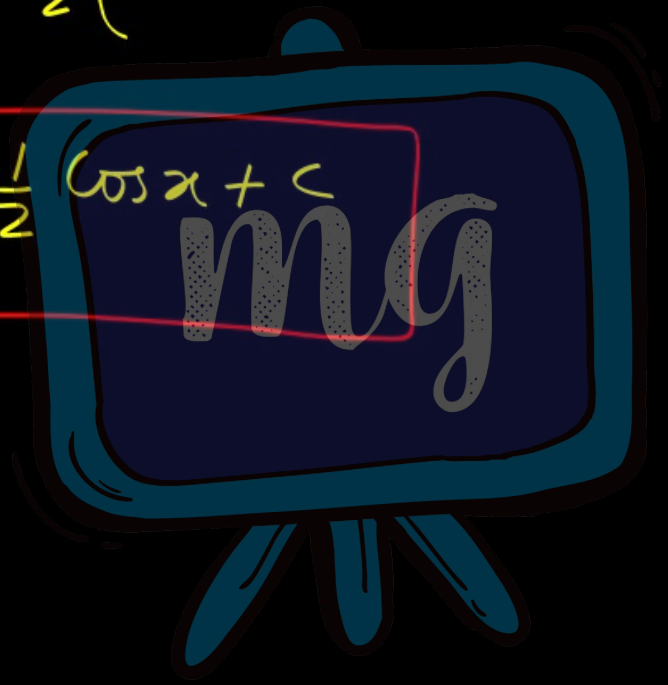
$= \frac{1}{2}\int [\sin 7x - \sin x] \, dx$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$= \frac{1}{2}\int \sin 7x \, dx - \frac{1}{2}\int \sin x \, dx$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}\left(-\cos x\right) + c$$

$$= \boxed{-\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + c}$$



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1 से 22 तक के प्रश्नों में प्रत्येक फलन का समाकलन ज्ञात कीजिए।

$$\int \cos 2x \cdot \cos 4x \cdot \cos 6x \, dx$$

$$= \frac{1}{2}\int (2\cos 2x \cos 4x)\cos 6x \, dx$$

Q3. $\cos 2x \cos 4x \cos 6x$

$$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2}\int [\cos 6x + \cos 2x]\cos 6x \, dx$$

$$= \frac{1}{2}\int [\cos^2 6x + \cos 2x \cdot \cos 6x] \, dx$$

$$= \frac{1}{2}\int \left[\cos^2 6x + \cos 2x \cdot \cos 6x\right] dx$$

$$= \frac{1}{2}\int \cos^2 6x\, dx + \frac{1}{2}\int \cos 2x \cdot \cos 6x\, dx$$

$$= \frac{1}{2}\int \frac{1+\cos 12x}{2}\, dx + \frac{1}{2}\cdot\frac{1}{2}\int 2\cos 2x \cdot \cos 6x\, dx$$

$$= \frac{1}{2}\int \frac{1+\cos 12x}{2}\, dx + \frac{1}{4}\int \left[\cos 8x + \cos 4x\right] dx$$

$\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow 2\cos^2\theta = 1 + \cos 2\theta$

$\Rightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \frac{1}{4}\int (1+\cos 12x)\,dx + \frac{1}{4}\int \cos 8x\,dx + \frac{1}{4}\int \cos 4x\,dx$$

$$= \frac{1}{4}\int 1\cdot dx + \frac{1}{4}\int \cos 12x\,dx + \frac{1}{4}\int \cos 8x\,dx + \frac{1}{4}\int \cos 4x\,dx$$

$$= \frac{1}{4}x + \frac{1}{4}\cdot\frac{\sin 12x}{12} + \frac{1}{4}\frac{\sin 8x}{8} + \frac{1}{4}\frac{\sin 4x}{4} + c$$

$$= \frac{x}{4} + \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + c$$

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1 से 22 तक के प्रश्नों में प्रत्येक फलन का समाकलन ज्ञात कीजिए।

Q4. $\sin^3(2x+1)$

$$\int \sin^3(2x+1)\,dx$$

$$= \int \sin^2(2x+1)\cdot \sin(2x+1)\,dx$$

$$= \int [1-\cos^2(2x+1)]\cdot \sin(2x+1)\,dx$$

$$= \int (1-t^2)\left(-\frac{dt}{2}\right)$$

$$= \frac{1}{2}\int (t^2-1)\,dt$$

माना $\cos(2x+1) = t$

$$-\sin(2x+1)\,(2)\,dx = dt$$

$$\Rightarrow \sin(2x+1)\,dx = -\frac{dt}{2}$$

$$= \frac{1}{2}\int (t^2 - 1)\, dt$$

$$= \frac{1}{2}\int t^2\, dt - \frac{1}{2}\int 1 \cdot dt$$

$$= \frac{1}{2}\frac{t^3}{3} - \frac{1}{2}t + C$$

$$= \frac{1}{6}\cos^3(2x+1) - \frac{1}{2}\cos(2x+1) + C$$

