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# कक्षा – 12

# गणित

## समाकलन

## अध्याय – 7

## भाग - 12

## प्रश्नावली 7.6

By : Rajendra Yadav Sir

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# उदाहरण –17

➤ $\int x \cos x \, dx$ का मान ज्ञात कीजिए ।

$$\int f(x) \cdot g(x) \, dx = f(x) \int g(x) \, dx - \int \left[ f'(x) \cdot \int g(x) \cdot dx \right] dx$$

माना $f(x) = x$ व $g(x) = \cos x$.

$$\int x \cos x \, dx = x \int \cos x \, dx - \int \left[ 1 \cdot \int \cos x \, dx \right] dx$$

$$= x(\sin x) - \int \sin x \, dx + c$$

$$= x \sin x - (-\cos x) + c$$

$$\boxed{\int x \cos x \, dx = x \sin x + \cos x + c}$$

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# उदाहरण –18

➤ $\int \log x \, dx$ ज्ञात कीजिए।

$\int \log x \cdot 1 \, dx$ माना $f(x) = \log x$ व $g(x) = 1$

$$\int f(x) g(x) \, dx = f(x) \int g(x) \, dx - \int [f'(x) \cdot \int g(x) \, dx] \, dx$$

$$\int \log x \cdot 1 \, dx = \log x \int 1 \cdot dx - \int \left[\frac{1}{x} \int 1 \cdot dx\right] dx$$

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$$\int \log x \cdot 1 \, dx = \log x \cdot x - \int \left(\frac{1}{x} \cdot x\right) dx + c$$

$$= x \log x - \int 1 \cdot dx + c$$

$$\int \log x \, dx = x \log x - x + c$$

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# उदाहरण –19

➤ $\int xe^x\,dx$ ज्ञात कीजिए।

माना $f(x) = x$ व $g(x) = e^x$

$$\int f(x)\cdot g(x)\,dx = f(x)\int g(x)\,dx - \int\left[f'(x)\cdot\int g(x)\,dx\right]dx$$

$$\int xe^x\,dx = x\int e^x\,dx - \int\left[1\cdot\int e^x\,dx\right]dx$$

$$= xe^x - \int e^x\,dx + c$$

$$= xe^x - e^x + c$$

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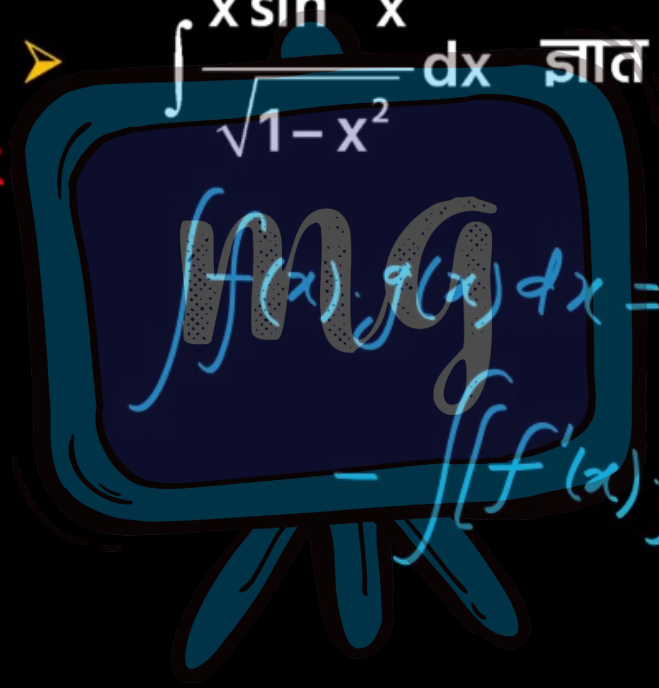
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# उदाहरण –20

➤ $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ ज्ञात कीजिए ।

माना $f(x) = \sin^{-1} x$

व $g(x) = \frac{x}{\sqrt{1-x^2}}$



$$\int f(x) \cdot g(x)\, dx = f(x) \int g(x)\, dx - \int [f'(x) \cdot \int g(x)\, dx]\, dx$$

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यहाँ $g(x) = \dfrac{x}{\sqrt{1-x^2}}$

माना $1-x^2 = t^2$

$\Rightarrow -2x\,dx = 2t\,dt$

$\Rightarrow x\,dx = -t\,dt$

अब, $\int g(x)\,dx = \int \dfrac{x}{\sqrt{1-x^2}}\,dx$

$= \int \dfrac{-t\,dt}{t}$

$= -\int dt$

$= -t + c$

$\Rightarrow \int g(x)\,dx = -t + c$

$= -\sqrt{1-x^2} + c$

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अब, $$\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}}\,dx = \sin^{-1}x \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$- \int \left[ \frac{d}{dx}\sin^{-1}x \cdot \int \frac{x}{\sqrt{1-x^2}}\,dx \right] dx$$

$$= \sin^{-1}x\left(-\sqrt{1-x^2}\right) - \int \frac{1}{\sqrt{1-x^2}} \cdot d\left(-\sqrt{1-x^2}\right) dx$$

$$= -\sin^{-1}x\left(\sqrt{1-x^2}\right) + \int 1 \cdot dx$$

$$= -\sin^{-1}x \cdot \sqrt{1-x^2} + x + c$$



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# उदाहरण –21

➤ $\int e^x \sin x \, dx$ ज्ञात कीजिए ।

माना $f(x) = e^x$ व $g(x) = \sin x$

$$\int f(x) \cdot g(x) \, dx = f(x) \int g(x) \, dx - \int [f'(x) \cdot \int g(x) \, dx] \, dx$$

$$\int e^x \sin x \, dx = e^x \int \sin x \, dx - \int [e^x \int \sin x \, dx] \, dx$$

$$= e^x (-\cos x) - \int e^x (-\cos x) \, dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx + c$$



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# उदाहरण –22

ज्ञात कीजिए ।

(i) $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$

$\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + c$

यहाँ माना $f(x)=\tan^{-1}x$

$\Rightarrow f'(x)=\frac{1}{1+x^2}$

$$\int e^x \left( \tan^{-1} x + \frac{1}{1+x^2} \right) dx = e^x \cdot \tan^{-1} x + c$$



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# उदाहरण –22

ज्ञात कीजिए ।

(ii) $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$

यहाँ $\frac{x^2+1}{(x+1)^2} = \frac{x^2+1+1-1}{(x+1)^2}$

$= \frac{x^2-1+2}{(x+1)^2}$

$= \frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}$

$= \frac{(x+1)(x-1)}{(x+1)^2} + \frac{2}{(x+1)^2}$

$$\frac{x^2+1}{(x+1)^2} = \frac{(x-1)}{(x+1)} + \frac{2}{(x+1)^2}$$

अतः, $\int \frac{(x^2+1)e^x}{(x+1)^2}\,dx = \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$

माना $f(x) = \frac{x-1}{x+1}$ $\Rightarrow f'(x) = \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$

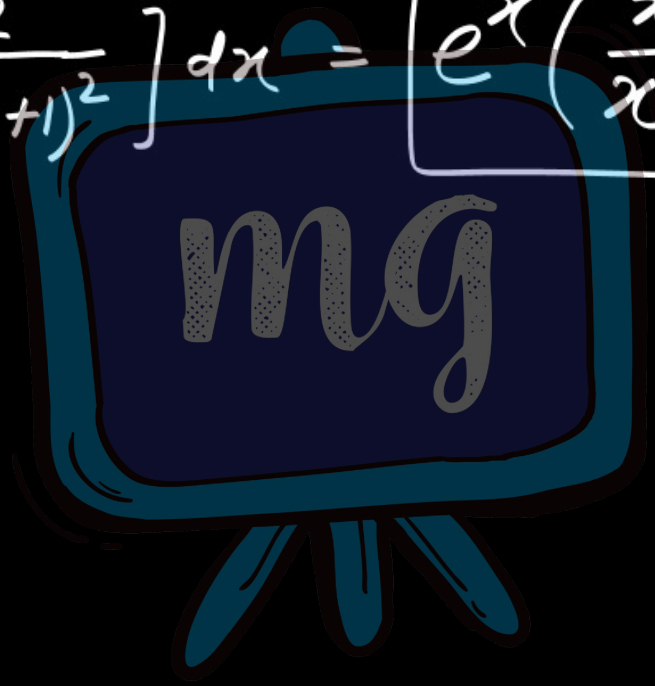
$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$

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अत:

$$\int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx = \boxed{e^x\left(\frac{x-1}{x+1}\right) + c}$$



# प्रश्नावली 7.6

➤ 1 से 22 तक के प्रश्नों के फलनों का समाकलन कीजिए।

**Q1.** x sin x

$\int x \sin x \, dx$

माना $f(x) = x$

व $g(x) = \sin x$

$\int f(x) \cdot g(x) \, dx = f(x) \int g(x) \, dx$

$- \int [f'(x) \int g(x) \, dx] \, dx$

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$$\int x \sin x \, dx = x \int \sin x \, dx - \int \left[ 1 \cdot \int \sin x \, dx \right] dx$$

$$= x(-\cos x) - \int (-\cos x) \, dx$$

$$= -x \cos x + \int \cos x \, dx$$

$$= -x \cos x + \sin x + C$$

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# प्रश्नावली 7.6

➤ 1 से 22 तक के प्रश्नों के फलनों का समाकलन कीजिए।

**Q2.** $x \sin 3x$

माना $f(x) = x$ व $g(x) = \sin 3x$

$$\int f(x) \cdot g(x)\, dx = f(x) \int g(x)\, dx - \int [f'(x) \int g(x)\, dx]\, dx$$

$$\int x \sin 3x\, dx = x \int \sin 3x\, dx - \int [1 \cdot \int \sin 3x\, dx]\, dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int \left[\frac{-\cos 3x}{3}\right] dx$$

$$= -\frac{x}{3}\cos 3x + \frac{1}{3}\left(\frac{\sin 3x}{3}\right) + c$$

$$= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$$

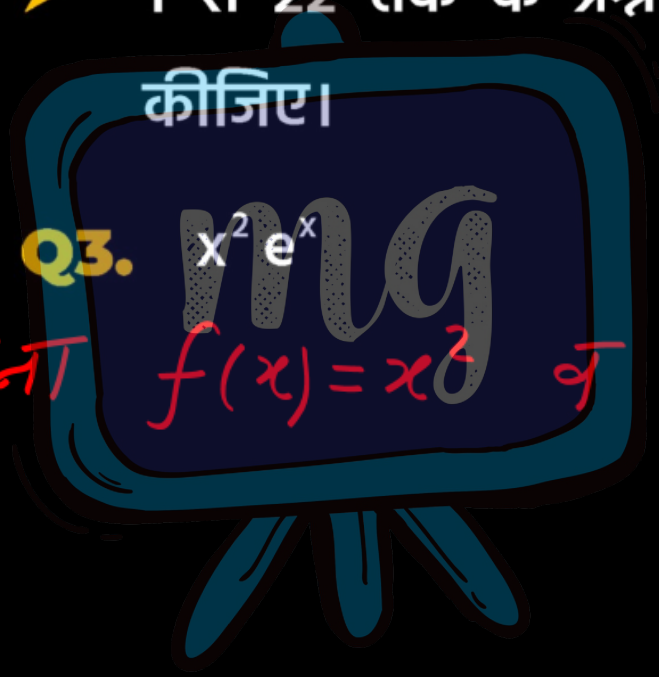
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# प्रश्नावली 7.6

➤ 1 से 22 तक के प्रश्नों के फलनों का समाकलन कीजिए।

**Q3.** $x^2 e^x$



माना $f(x) = x^2$ व $g(x) = e^x$