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कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 10

प्रश्नावली 5.4

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उदाहरण–24 : क्या यह सत्य है कि x के सभी वास्तविक मानों के लिए $x = e^{\log x}$ है?

माना $y = e^{\log x}$

दोनों तरफ log लेने पर :

$\log y = \log e^{\log x}$

$\log y = \log x \cdot \log e$

$\log y = \log x$

$\Rightarrow y = x$

$y > 0$ तथा $x > 0$

$x = e^{\log x}$

$\log_a b = b \log a$

$\log_a a = 1$

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उदाहरण–25 : x के सापेक्ष निम्नलिखित का अवकलन कीजिए:

(i) e^{-x}

(ii) $\sin(\log x),\ x > 0$

(iii) $\cos^{-1}(e^x)$

(iv) $e^{\cos x}$

$\frac{d}{dx} e^x = e^x$

(i) $y = e^{-x}$

$\frac{dy}{dx} = \frac{d}{dx}(e^{-x})$

$\frac{dy}{dx} = e^{-x} \frac{d}{dx}(-x)$

$\frac{dy}{dx} = e^{-x}(-1) = -e^{-x}$

$\boxed{\frac{dy}{dx} = -e^{-x}}$

(ii) $y = \sin(\log x)$

$$\frac{dy}{dx} = \cos(\log x) \frac{d}{dx}(\log x)$$

$$\frac{dy}{dx} = \cos(\log x) \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} \cdot \cos(\log x)$$

(iii) $y = \cos^{-1}(e^x)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-e^{2x}}} \frac{d}{dx}(e^x)$$

$$\boxed{\frac{dy}{dx} = \frac{-e^x}{\sqrt{1-e^{2x}}}}$$

(iv) $y = e^{\cos x}$

$$\frac{dy}{dx} = e^{\cos x} \frac{d}{dx} \cos x$$

$$\boxed{\frac{dy}{dx} = -\sin x \cdot e^{\cos x}}$$

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निम्नलिखित का x के सापेक्ष अवकलन कीजिए:

1. $\frac{e^x}{\sin x}$

$$y = \frac{e^x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x \cdot e^x - e^x \cdot \cos x}{\sin^2 x}$$

$$\boxed{\frac{dy}{dx} = \frac{(\sin x - \cos x)\, e^x}{\sin^2 x}}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v u' - u v'}{v^2}$$

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2. $e^{\sin^{-1}x}$

$y = e^{\sin^{-1}x}$

$\frac{dy}{dx} = e^{\sin^{-1}x} \frac{d}{dx} \sin^{-1}x$

$\frac{dy}{dx} = e^{\sin^{-1}x} \left[\frac{1}{\sqrt{1-x^2}}\right]$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} e^{\sin^{-1}x}}$

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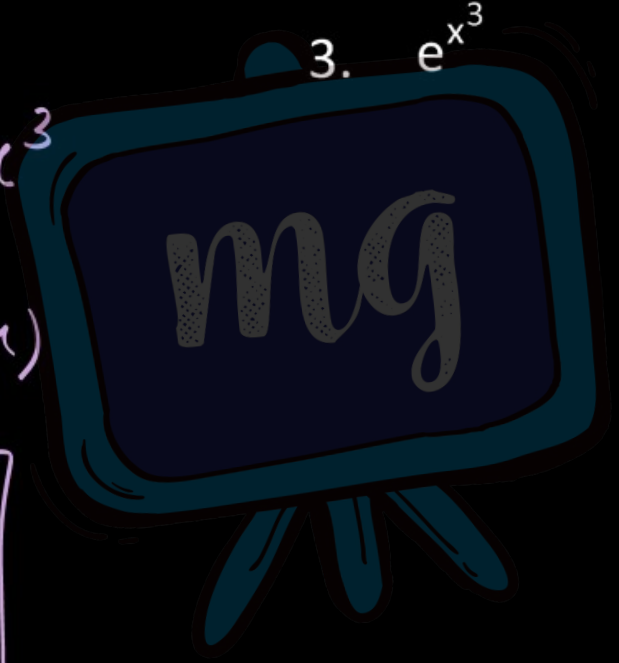
3. e^{x^3}

$$y = e^{x^3}$$

$$\frac{dy}{dx} = e^{x^3} \frac{d}{dx} x^3$$

$$\frac{dy}{dx} = e^{x^3} \cdot (3x)$$

$$\boxed{\frac{dy}{dx} = 3x e^{x^3}}$$



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$y = \sin(\tan^{-1} e^{-x})$

4. $\sin(\tan^{-1} e^{-x})$

$$\frac{dy}{dx} = \cos(\tan^{-1} e^{-x}) \frac{d}{dx} \tan^{-1} e^{-x}$$

$$\frac{dy}{dx} = \cos(\tan^{-1} e^{-x}) \frac{1}{1+e^{-2x}} \frac{d}{dx} e^{-x}$$

$$\frac{dy}{dx} = \frac{-e^{-x}}{1+e^{-2x}} \cdot \cos(\tan^{-1} e^{-x})$$

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5. $\log(\operatorname{cosec} e^x)$

$y = \log(\operatorname{cosec} e^x)$

$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} e^x} \frac{d}{dx} \operatorname{cosec} e^x$

$\frac{d}{dx} \log x = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} e^x} \left[-\sin e^x \right] \frac{d}{dx} e^x$

$\frac{dy}{dx} = -e^x \cdot \tan e^x$

$\frac{dy}{dx} = -e^x \cdot \frac{\sin e^x}{\cos e^x}$

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6. $e^{x^1} + e^{x^2} + \ldots + e^{x^5}$

$$y = e^{x} + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$$

$$\frac{dy}{dx} = e^{x} + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}x^4 + e^{x^5}\frac{d}{dx}x^5$$

$$\frac{dy}{dx} = e^{x} + e^{x^2}(2x) + e^{x^3}(3x^2) + e^{x^4}(4x^3) + e^{x^5}(5x^4)$$

$$\boxed{\frac{dy}{dx} = e^{x} + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}}$$

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7. $\sqrt{e^{\sqrt{x}}}, x > 0$

$y = \sqrt{e^{\sqrt{x}}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \frac{d}{dx} e^{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \frac{d}{dx}\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$$\frac{dy}{dx} = \frac{1}{4} \frac{e^{\sqrt{x}}}{\sqrt{x} \cdot \sqrt{e^{\sqrt{x}}}}$$

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8. $\log(\log x), x > 1$

$y = \log(\log x)$

$\frac{dy}{dx} = \frac{1}{\log x} \frac{d}{dx} \log x$

$\frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x}$

$\boxed{\frac{dy}{dx} = \frac{1}{x \cdot \log x}}$

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$$y = \frac{\cos x}{\log x}$$

9. $\dfrac{\cos x}{\log x}, x > 0$

$$\frac{dy}{dx} = \frac{\log x(-\sin x) - \cos x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v u' - u v'}{v^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin x \cdot \log x - \frac{\cos x}{x}}{(\log x)^2}}$$

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10. $\cos\left(\log x + e^x\right), x > 0$

$$y = \cos(\log x + e^x)$$

$$\frac{dy}{dx} = -\sin(\log x + e^x)\,\frac{d}{dx}(\log x + e^x)$$

$$\frac{dy}{dx} = -\sin(\log x + e^x)\left(\frac{1}{x} + e^x\right)$$

$$\boxed{\frac{dy}{dx} = -\left(\frac{1}{x} + e^x\right)\cdot\sin(\log x + e^x)}$$