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कक्षा – 12

गणित

समाकलन

अध्याय – 7

भाग - 10

प्रश्नावली 7.5

By : Rajendra Yadav Sir

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उदाहरण –15

$\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi} d\phi$ का मान ज्ञात कीजिए।

माना $\sin\phi = y$

$\cos\phi \, d\phi = dy$

अतः $\int \frac{3y-2}{5-(1-y^2)-4y} dy = \int \frac{3y-2}{-5+y^2-4y} dy$

$= \int \frac{3y-2}{y^2-4y-5} dy$

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अब, $\int \frac{3y-2}{y^2-4y-5}\,dy = \int \frac{3y-2}{(y-5)(y+1)}\,dy$

अब, $\frac{3y-2}{(y-5)(y+1)} = \frac{A}{y-5} + \frac{B}{y+1}$

$\frac{3y-2}{(y-5)(y+1)} = \frac{A(y+1)+B(y-5)}{(y-5)(y+1)}$

$$3y-2 = A(y+1) + B(y-5)$$

$$\Rightarrow 3y-2 = Ay + A + By - 5B$$

$$\Rightarrow 3y-2 = (A+B)y + (A-5B)$$

तुलना करने पर:

$$A+B = 3$$

तथा $A-5B = -2$

$$\Rightarrow A-5B = -2$$
$$A+B = 3$$
$$-6B = -5$$

$$B = \frac{5}{6}$$

$$A+B = 3$$

$$A + \frac{5}{6} = 3$$

$$A = 3 - \frac{5}{6}$$

$$A = \frac{18-5}{6}$$

$$A = \frac{13}{6}$$

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अब,

$$\int \frac{3y-2}{(y-5)(y+1)}\,dy = \frac{13}{6}\int \frac{1}{y-5}\,dy + \frac{5}{6}\int \frac{1}{y+1}\,dy$$

$$= \frac{13}{6}\log|y-5| + \frac{5}{6}\log|y+1| + c$$

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उदाहरण –16

$\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$ ➤ $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$ का मान ज्ञात कीजिए।

$$= \int \frac{A}{x+2}dx + \int \frac{B}{x^2+1}dx$$

$$\Rightarrow \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{B}{x^2+1}$$

$$\Rightarrow \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+B(x+2)}{(x+2)(x^2+1)}$$

$$\Rightarrow \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+A+Bx+2B}{(x+2)(x^2+1)}$$

$$\Rightarrow \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+Bx+(A+2B)}{(x+2)(x^2+1)}$$

तुलना करने पर: $A=1$, $B=1$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{A}{x+2}\,dx + \int \frac{B}{x^2+1}\,dx$$

$$= \log|x+2| + \tan^{-1}x + C$$



प्रश्नावली 7.5

$$\int \frac{x}{(x+1)(x+2)}\,dx$$

➤ प्रश्न 1 से 21 तक के प्रश्नों में परिमेय फलनों का समाकलन कीजिए।

Q1. $\frac{x}{(x+1)(x+2)}$

माना

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

$$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{Ax + 2A + Bx + B}{(x+1)(x+2)}$$

$$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{(A+B)x + (2A+B)}{(x+1)(x+2)}$$

तुलना करने पर :

$A + B = 1$. $2A + B = 0$

$A + B = 1$

$2A + B = 0$

$-A = 1$

$\boxed{A = -1}$

$A + B = 1$

$-1 + B = 1$

$\boxed{B = 2}$

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अतः $\int \frac{x}{(x+1)(x+2)}\,dx = \int \frac{A}{x+1}\,dx + \int \frac{B}{x+2}\,dx$

$\Rightarrow \int \frac{x}{(x+1)(x+2)}\,dx = \int \frac{-1}{x+1}\,dx + \int \frac{2}{x+2}\,dx$

$= -\log|x+1| + 2\log|x+2| + c$

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प्रश्नावली 7.5

$$\int \frac{1}{x^2-9}\,dx$$

$$= \int \frac{1}{(x+3)(x-3)}\,dx$$

अब, $\frac{1}{(x+3)(x-3)}$

$$= \frac{A}{x+3} + \frac{B}{x-3}$$

➤ प्रश्न 1 से 21 तक के प्रश्नों में परिमेय फलनों का समाकलन कीजिए।

Q2. $\frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$

$$\frac{1}{(x+3)(x-3)} = \frac{A(x-3)+B(x+3)}{(x+3)(x-3)}$$

$$\Rightarrow \frac{1}{(x+3)(x-3)} = \frac{Ax-3A+Bx+3B}{(x+3)(x-3)}$$

$$\Rightarrow \frac{1}{(x+3)(x-3)} = \frac{(A+B)x-3A+3B}{(x+3)(x-3)}$$

तुलना करने पर : $A+B=0$ $\quad -3A+3B=1$

$\Rightarrow 3A+3B=0$

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अतः, $-3A + 3B = 1$

$3A + 3B = 0$

$6B = 1$

$B = \frac{1}{6}$

$A + B = 0$

$A + \frac{1}{6} = 0$

$A = -\frac{1}{6}$

अतः $\int \frac{1}{(x+3)(x-3)} dx = \int \frac{A}{x+3} dx + \int \frac{B}{x-3} dx$

$$\int \frac{1}{(x+3)(x-3)}\,dx = -\frac{1}{6}\int \frac{1}{x+3}\,dx + \frac{1}{6}\int \frac{1}{x-3}\,dx$$

$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + c$$

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प्रश्नावली 7.5

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$$ ➤

प्रश्न 1 से 21 तक के प्रश्नों में परिमेय फलनों का समाकलन कीजिए।

Q3. $\dfrac{3x-1}{(x-1)(x-2)(x-3)}$

माना

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

तुलना करने पर :

$$3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$$

यदि $x=1$ हो :

$$3-1 = A(-1)(-2)$$

$$\Rightarrow 2 = 2A$$

$$\Rightarrow \boxed{A=1}$$

यदि $x=2$ हो :

$$6-1 = B(1)(-1)$$

$$5 = -B$$

$$\boxed{B=-5}$$

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यदि $x = 3$ देते:

$9 - 1 = C(2)(1)$

$\Rightarrow 8 = 2C$

$\Rightarrow C = \frac{8}{2} = 4$

$\Rightarrow \boxed{C = 4}$

अब, $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

$= \int \frac{A}{x-1}\,dx + \int \frac{B}{x-2}\,dx + \int \frac{C}{x-3}\,dx$

$= \int \frac{1}{x-1}\,dx - 5\int \frac{1}{x-2}\,dx + 4\int \frac{1}{x-3}\,dx$

$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$