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कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 8

केशव शर्मा

प्रश्नावली 8.3

Q.4 निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(iv) $$\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$$

$$\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$$

RHS.

$$= \frac{\sin^2 A}{1-\cos A}$$

$$\left\{ \begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \sin^2\theta = 1-\cos^2\theta \end{array} \right\}$$

$$= \frac{1^2-\cos^2 A}{1-\cos A}$$

$$\{a^2-b^2=(a+b)(a-b)\}$$

$$= \frac{(1+\cos A)\cancel{(1-\cos A)}}{\cancel{1-\cos A}}$$

$$= 1+\cos A$$

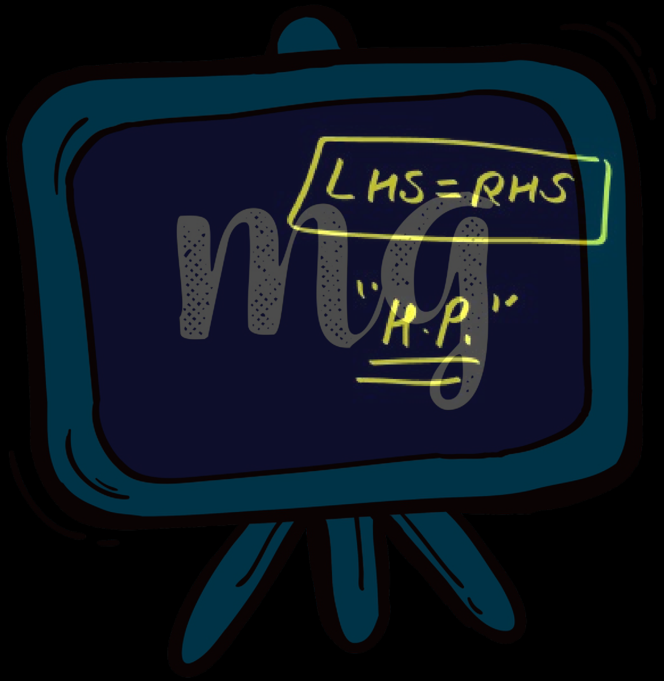
LHS.

$$\frac{1+\sec A}{\sec A}$$

$$\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{(1+\cos A)\times\cancel{\cos A}}{\cancel{\cos A}}$$

$$= 1+\cos A$$


LHS=RHS
"H.P."

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(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$\frac{1}{\sin}$ $\frac{\cos}{\sin}$

हल:- LHS

$\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

अंश व हर में sin A का भाग देने पर

$= \dfrac{\dfrac{\cos A - \sin A + 1}{\sin A}}{\dfrac{\cos A + \sin A - 1}{\sin A}}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\cancel{\sin A}}{\cancel{\sin A}} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\cancel{\sin A}}{\cancel{\sin A}} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A + 1 - \operatorname{cosec} A}$$

$$\{1 = \operatorname{cosec}^2 A - \cot^2 A\}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec}A + \cot A) - [(\operatorname{cosec}A + \cot A)(\operatorname{cosec}A - \cot A)]}{\cot A + 1 - \operatorname{cosec}A}$$

$$= \frac{(\operatorname{cosec}A + \cot A)[1 - (\operatorname{cosec}A - \cot A)]}{\cot A + 1 - \operatorname{cosec}A}$$

$$= \frac{(\operatorname{cosec}A + \cot A)[1 - \operatorname{cosec}A + \cot A]}{\cot A + 1 - \operatorname{cosec}A}$$

$$= \frac{(\operatorname{cosec}A + \cot A)\cancel{(\cot A + 1 - \operatorname{cosec}A)}}{\cancel{\cot A + 1 - \operatorname{cosec}A}}$$

$$= \operatorname{cosec}A + \cot A$$

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(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

हल:- LHS $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

हर का परिमेयकरण करने पर

$= \sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

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$$= \sqrt{\frac{(1+\sin A)^2}{1^2-\sin^2 A}}$$

$$\sin^2\theta+\cos^2\theta=1$$

$$\left\{ 1-\sin^2\theta=\cos^2\theta \right\}$$

$$= \frac{\sqrt{(1+\sin A)^2}}{\sqrt{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\sqrt{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \underline{RHS}$$

$$\boxed{LHS = RHS}$$

"H.P."

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \boxed{\tan\theta}$

$\sin\theta \times \sin\theta \times \cancel{\sin\theta}$

हल:- LHS

$$= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= \tan\theta\left[\frac{1 - 2(1 - \cos^2\theta)}{2\cos^2\theta - 1}\right]$$

$$= \tan\theta\left[\frac{1 - 2 + 2\cos^2\theta}{2\cos^2\theta - 1}\right]$$

$$= \tan\theta\left[\frac{\cancel{2\cos^2\theta - 1}}{\cancel{2\cos^2\theta - 1}}\right]$$

$$= \tan\theta = \text{RHS}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

LHS. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$\sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A$

$+ 2\cos A \sec A$

$= \sin^2 A + \cos^2 A + 2\cancel{\sin A}\frac{1}{\cancel{\sin A}} + 2\cancel{\cos A} \times \frac{1}{\cancel{\cos A}}$

$+ \operatorname{cosec}^2 A + \sec^2 A$

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$$= 1 + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$$

$$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$$

$$= \underline{7 + \tan^2 A + \cot^2 A} = \underline{RHS}$$

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(ix) $(\operatorname{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

$\underline{\text{LHS.}}$ $(\operatorname{cosec}A - \sin A)(\sec A - \cos A)$

$$\left(\frac{1}{\sin A} - \frac{\sin A}{1}\right)\left(\frac{1}{\cos A} - \frac{\cos A}{1}\right)$$

$$\left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$\frac{\cos^{\cancel{2}} A}{\cancel{\sin A}} \cdot \frac{\sin^{\cancel{2}} A}{\cancel{\cos A}}$$

$$= \sin A \cos A$$

RHS. $= \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \cos A \sin A$$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$\cot A = \dfrac{1}{\tan A}$