कक्षा - 12

Chapter - 7

समाकलन

निश्चित समाकलनों के गुणधर्म

विविध प्रश्नावली

Part - 23

Lakhan Saini

1. $\dfrac{1}{x^2(x^4+1)^{\frac{3}{4}}}$

हल :- माना $I = \int \dfrac{1}{x^2(x^4+1)^{3/4}}\,dx$

$= \int \dfrac{dx}{x^2(x^4)^{3/4}\left(1+\frac{1}{x^4}\right)^{3/4}}$

$= \int \dfrac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

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माना $1+\frac{1}{x^4}=t^4$

$-\frac{4}{x^5}=4t^3\frac{dt}{dx}$

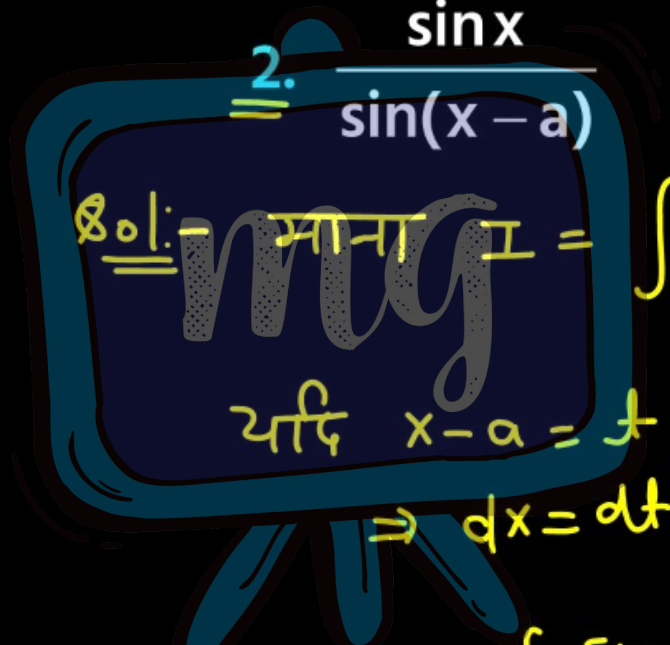
$\Rightarrow \frac{dx}{x^5}=-t^3dt$

$\therefore I=\int\frac{-t^3dt}{(t^4)^{3/4}}=-\int\frac{t^3dt}{t^3}$

$=-t+C$

$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$

2. $\dfrac{\sin x}{\sin(x-a)}$



हल :- माना $I = \int \dfrac{\sin x\, dx}{\sin(x-a)}$

यदि $x - a = t \Rightarrow x = a + t$

$\Rightarrow dx = dt$

$\therefore \quad I = \int \dfrac{\sin(a+t)\, dt}{\sin t}$

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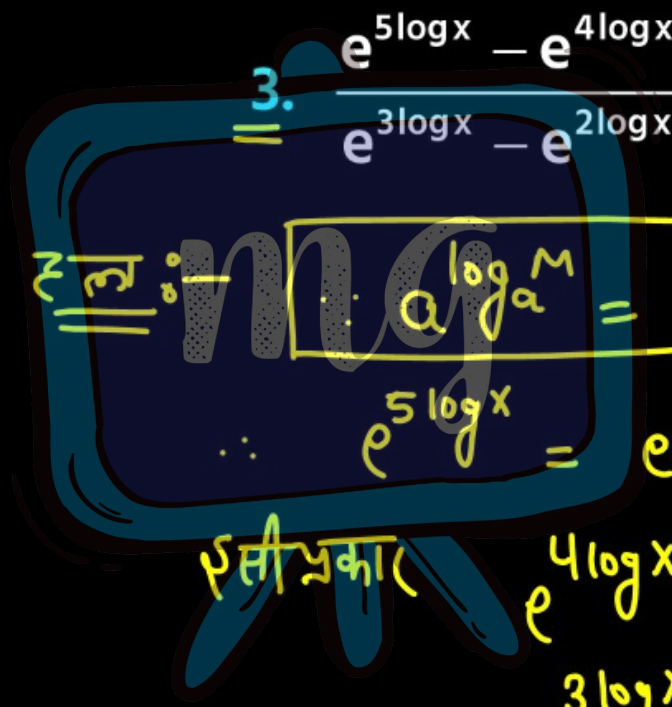
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$$= \int \frac{\sin a \cos t + \cos a \sin t}{\sin t} \, dt$$

$$= \sin a \int \cot t \, dt + \cos a \int dt$$

$$= \sin a \log|\sin t| + \cos a \, t + C$$

$$= \sin a \log|\sin(x-a)| + \cos a \, (x-a) + C$$



3. $\dfrac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

हल :- $\because a^{\log_a M} = M$ V.V.I.

$\therefore e^{5\log x} = e^{\log x^5} = x^5$ $[a\log b = \log b^a]$

इसी प्रकार $e^{4\log x} = x^4$

$e^{3\log x} = x^3$

$e^{2\log x} = x^2$

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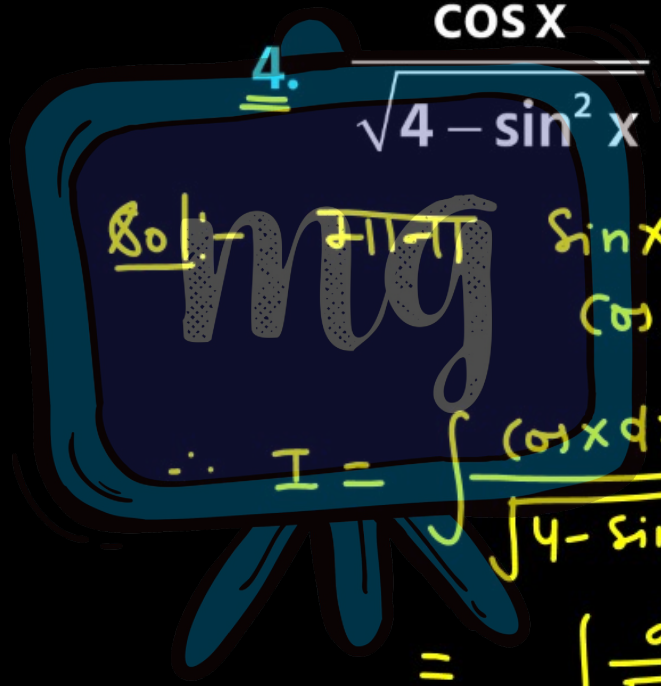
$$\therefore \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\, dx$$

$$= \int \frac{x^5 - x^4}{x^3 - x^2}\, dx$$

$$= \int \frac{x^4(x-1)}{x^2(x-1)}\, dx$$

$$= \int x^2\, dx = \frac{x^3}{3} + C$$

4. $\dfrac{\cos x}{\sqrt{4-\sin^2 x}}$



Sol:- माना $\sin x = t$

$\cos x\,dx = dt$

$\therefore I = \int \dfrac{\cos x\,dx}{\sqrt{4-\sin^2 x}}$

$= \int \dfrac{dt}{\sqrt{(2)^2 - t^2}} = \sin^{-1}\left(\dfrac{t}{2}\right) + C$

$= \sin^{-1}\left(\dfrac{\sin x}{2}\right) + C$

5. $\dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

Sol:- $\because \dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

$= \dfrac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x \cos^2 x}$

$= \dfrac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x]}{1 - 2\sin^2 x \cos^2 x}$

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$$= \frac{(\sin^2 x - \cos^2 x)(1)(1 - 2\sin^2 x \cos^2 x)}{(1 - 2\sin^2 x \cos^2 x)}$$

$$= -\cos 2x$$

$$\therefore \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx = -\int \cos 2x\, dx$$

$$= -\frac{1}{2}\sin 2x + c$$

V.V.I.

6. $\dfrac{1}{\cos(x+a)\cos(x+b)}$

Sol:-

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[\overset{A}{(x+a)}-\overset{B}{(x+b)}]}{\cos(x+a)\cos(x+b)}\right]$$

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$$= \frac{1}{\sin(a-b)} \left[\frac{\sin(x+a)\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)} \right]$$

$$= \frac{1}{\sin(a-b)} \left[\tan(x+a) - \tan(x+b) \right]$$

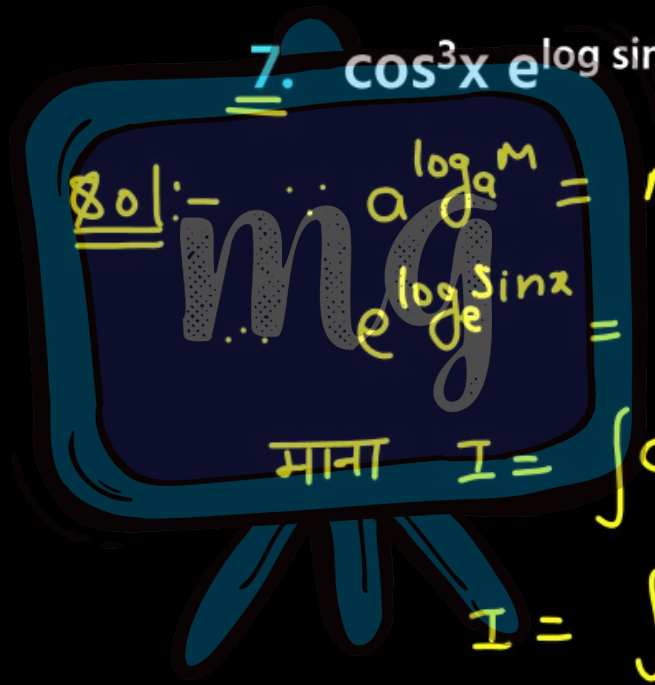
$$\therefore \int \frac{1}{\cos(x+a)\cos(x+b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \left[\tan(x+a) - \tan(x+b) \right] dx$$

$$= \frac{1}{\sin(a-b)} \left[-\log|\cos(x+a)| + \log|\cos(x+b)| \right] + c$$

$$= \frac{1}{\sin(a-b)} \log \left| \frac{\cos(x+b)}{\cos(x+a)} \right| + c$$

7. $\cos^3 x\, e^{\log \sin x}$



हल :- $a^{\log_a M} = M$

$e^{\log_e \sin x} = \sin x$

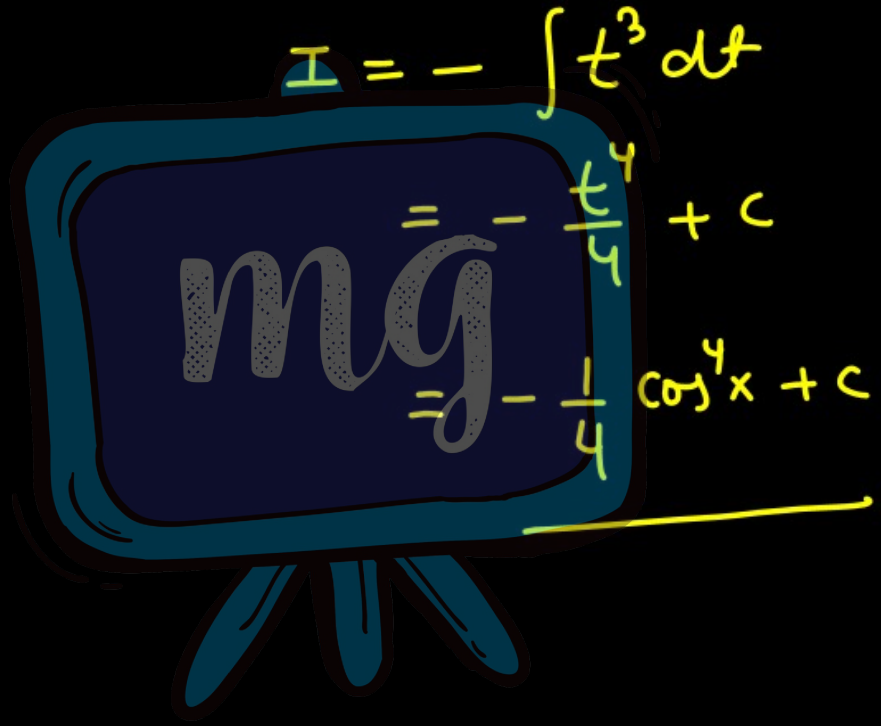
माना $I = \int \cos^3 x\, e^{\log \sin x}\, dx$

$I = \int \cos^3 x \sin x\, dx$

यदि $\cos x = t \Rightarrow \sin x\, dx = -dt$

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8. $f'(ax + b)\,[f(ax + b)]^n$

हल :- माना $I = \int f'(ax+b)\,[f(ax+b)]^n dx$

यदि $f(ax+b) = t$

$a\,f'(ax+b)\,dx = dt$

$f'(ax+b)\,dx = \frac{1}{a}\,dt$

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$$\therefore \quad I = \frac{1}{a}\int t^n \, dt$$

$$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right] + c$$

$$= \frac{1}{a(n+1)}\left[f(ax+b)\right]^n + c$$

9. $\frac{2+\sin 2x}{1+\cos 2x} e^x$

Sol:- $I = \int \left(\frac{2+\sin 2x}{1+\cos 2x}\right) e^x dx$

$\therefore \int e^x (f(x) + f'(x)) dx = e^x f(x) + C$

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$$\therefore \quad \frac{2+\sin 2x}{1+\cos 2x}$$

$$= \frac{2+2\sin x\cos x}{1+2\cos^2 x-1}$$

$$= \frac{2(1+\sin x\cos x)}{2\cos^2 x}$$

$$= \sec^2 x + \tan x$$

$$\therefore I = \int e^x(\tan x + \sec^2 x)\,dx$$

$$= e^x \tan x + c$$

10. $\int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

Sol:- $\frac{1-\sin x}{1-\cos x}$

$= \frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{1-(1-2\sin^2\frac{x}{2})}$

$= \frac{1-2\sin\frac{x}{2}\cos x/2}{2\sin^2\frac{x}{2}}$

$= \frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}$

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$$= \left(-\cot\frac{x}{2}\right) + \frac{1}{2}\operatorname{cosec}^2 x/2$$

यदि $f(x) = -\cot x/2$

$f'(x) = \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$

$$\therefore I = \int e^x (f(x) + f'(x))dx$$

$$= e^x f(x) + c$$

$$= -e^x \cot\frac{x}{2} + c$$

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$$\therefore \int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$$

$$= \left[-e^x \cot \frac{x}{2} \right]_{\pi/2}^{\pi}$$

$$= -e^{\pi} \cot \frac{\pi}{2} + e^{\pi/2} \cot \pi/4$$

$$= e^{\pi/2}$$

$$=$$

11. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4\sin^2 x}$

हल :- माना $I = \int \frac{\cos^2 x \, dx}{\cos^2 x + 4\sin^2 x}$

$I = \int \frac{\cos^2 x \, dx}{4 - 3\cos^2 x}$ $[\sin^2 x = 1 - \cos^2 x]$

$I = \frac{-1}{3} \int \frac{-3\cos^2 x + 4 - 4}{4 - 3\cos^2 x} dx$

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$$\Rightarrow I = -\frac{1}{3}\int\left(1 - \frac{4}{4-3\cos^2 x}\right)dx$$

$$I = -\frac{1}{3}\int dx + \frac{4}{3}\int\frac{dx}{4-3\cos^2 x}$$

I_1

यहाँ $I_1 = \frac{4}{3}\int\frac{\sec^2 x\,dx}{4(1+\tan^2 x)-3}$

यदि $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$I_1 = \frac{4}{3}\int\frac{dt}{4(1+t^2)-3}$$

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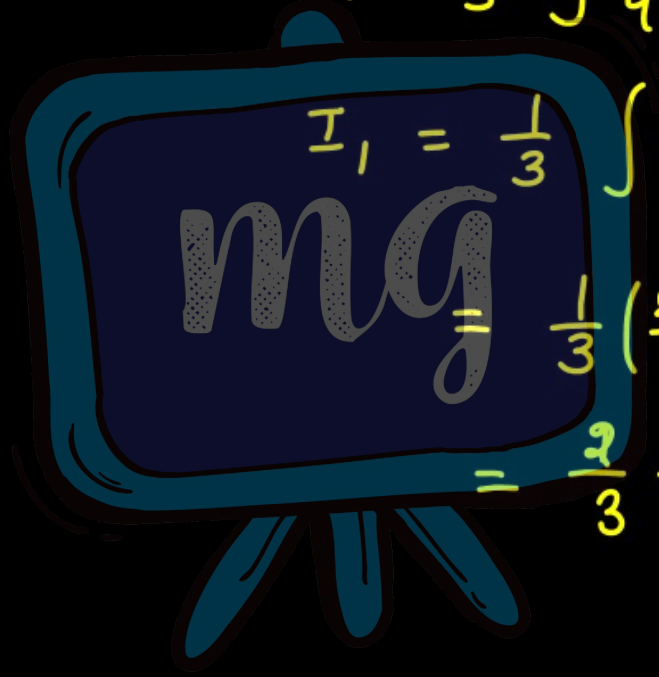
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$$I_1 = \frac{4}{3}\int \frac{dt}{4t^2+1}$$

$$I_1 = \frac{1}{3}\int \frac{dt}{t^2+\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{3}\left(\frac{2}{1}\right)\tan^{-1}(2t) + C$$

$$= \frac{2}{3}\tan^{-1}(2\tan\theta) + C$$



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$$\therefore I = \frac{-1}{3}\int dx + \frac{2}{3}\tan^{-1}(2\tan x) + C$$

$$\therefore \int_0^{\pi/2} \frac{\cos^2 x\, dx}{\cos^2 x + 4\sin^2 x} = \left[-\frac{x}{3} + \frac{2}{3}\tan^{-1}(2\tan x)\right]_0^{\pi/2}$$

$$= -\frac{\pi}{6} + \frac{2}{3}\tan^{-1}(\infty) - 0$$

$$= -\frac{\pi}{6} + \frac{2}{3}\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{6}$$

12. $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

Sol:- $I = \int_0^1 \frac{\left[\sqrt{1+x}+\sqrt{x}\right]}{1+x-x} dx$

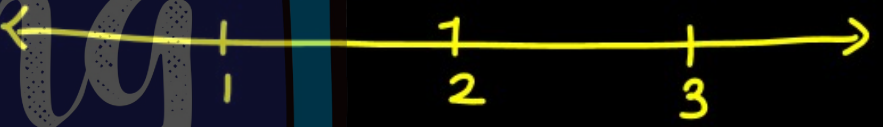
$= \frac{2}{3}\left[(1+x)^{3/2} + (x)^{3/2}\right]_0^1$

$= \frac{2}{3}\left((2)^{3/2} + 1\right)$

$= \frac{2}{3}(2\sqrt{2}+1)$

13. $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

$2x-3-x+3$

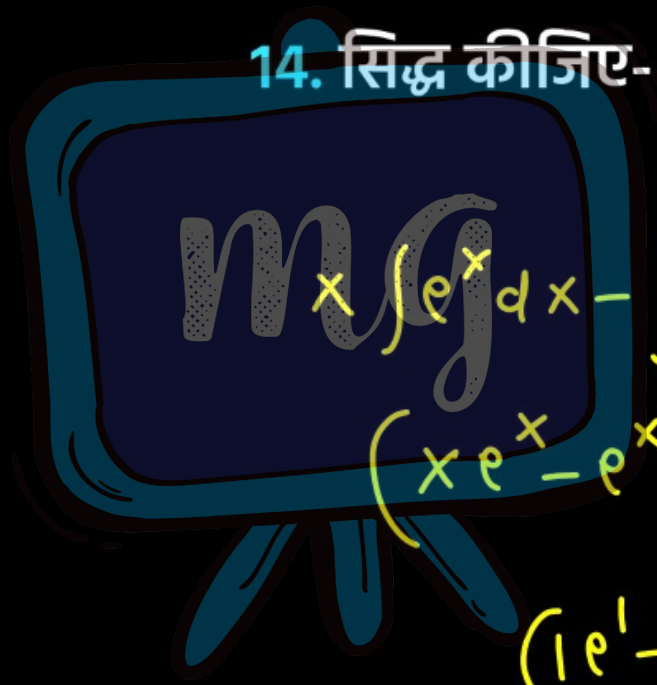


$x-1-x+2-x+3$

$\int_1^2 -x+4 + \int_2^3 x\,dx$

$\int_3^4 (3x-6)dx$

14. सिद्ध कीजिए- $\int_0^1 xe^x dx = 1$



$$x\int e^x dx - \int\left\{\frac{d}{dx}x\int e^x dx\right\}dx$$

$$\left(xe^x - e^x\right)_0^1$$

$$(1e^1 - e^1) - (0e^0 - e^0)$$

$$= 1 = RHS$$

15. सिद्ध कीजिए- $\int_0^{\frac{\pi}{2}} \sin^3 x dx = \frac{2}{3}$



$$\int_0^{\pi/2} \sin^2 x \sin x dx$$

$$\int_0^{\pi/2} (1-\cos^2 x) \sin x dx$$

$\cos x = t \quad - \sin x dx = dt$

$$-\int_1^0 (1-t^2) dt$$

16. $\int \frac{dx}{e^x + e^{-x}}$ बराबर है-

(A) $\tan^{-1}(e^x) + C$

(B) $\tan^{-1}(e^{-x}) + C$

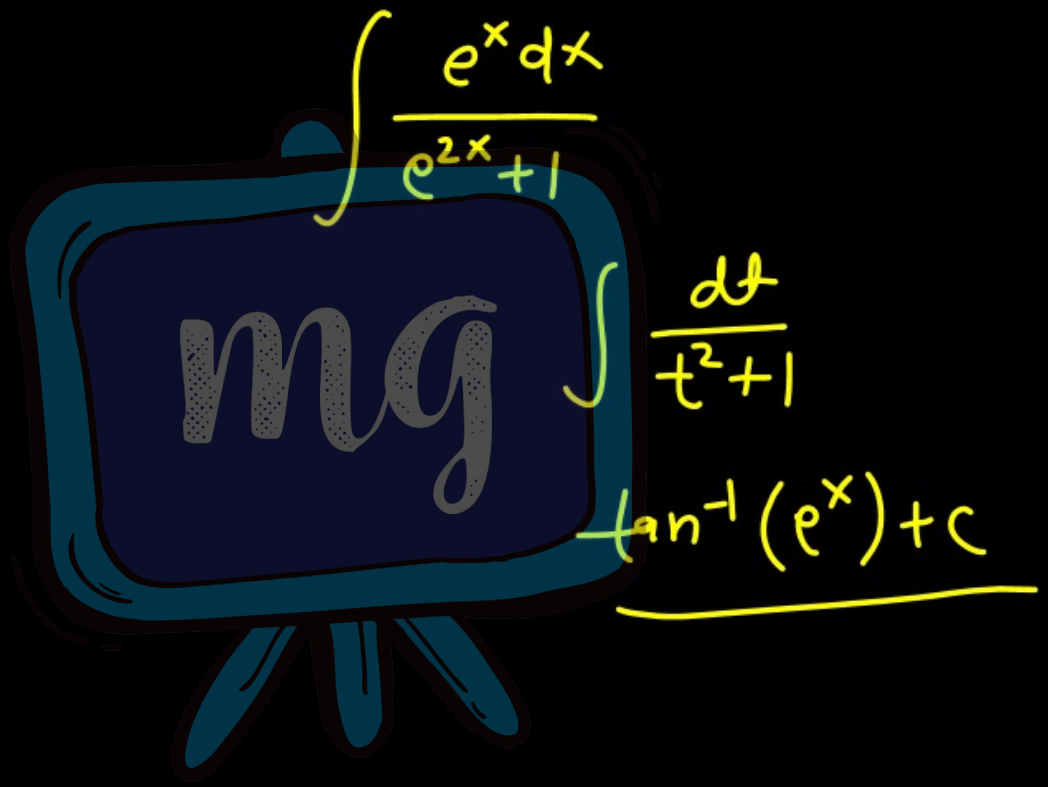
(C) $\log(e^x - e^{-x}) + C$

(D) $\log(e^x + e^{-x}) + C$


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17. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ बराबर है-

$\cos 2x \rightarrow \cos^2 x - \sin^2 x$

$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

(A) $\frac{-1}{\sin x + \cos} + C$ (B) $\frac{1}{(\sin x + \cos x)^2}$

(C) $\log|\sin x + \cos x| + C$

(D) $\log|\sin x - \cos x| + C$