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ई-पाठशाला कार्यक्रम

Class 12th

MATHS

अवकल समीकरण

02

RAPID REVISION



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$$x^2+3x+2=0$$



$$y^3+3y-11=0$$

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कोटि (order)

घात (degree)

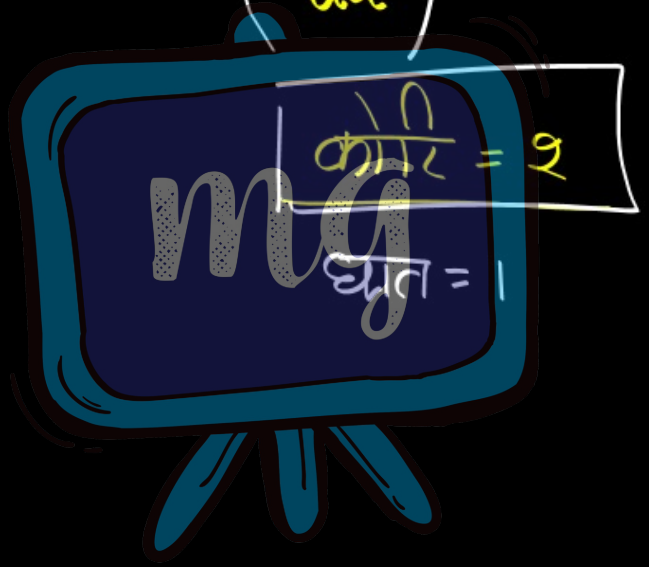
↳ सबसे ज्यादा बार अवकलन



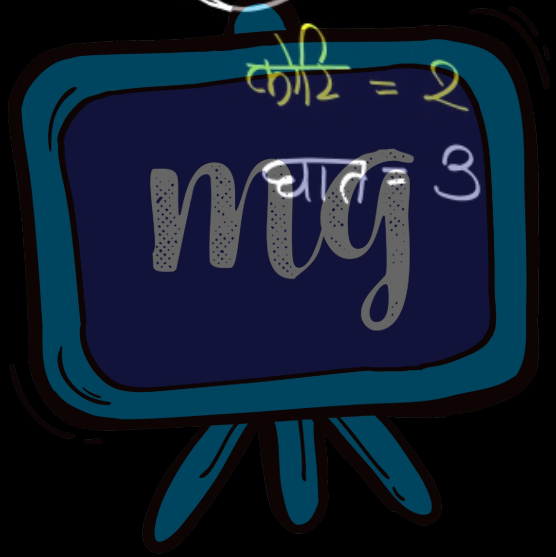
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$$\left(\frac{d^2y}{dx^2}\right)^1 = \cos 3x + \sin 3x$$

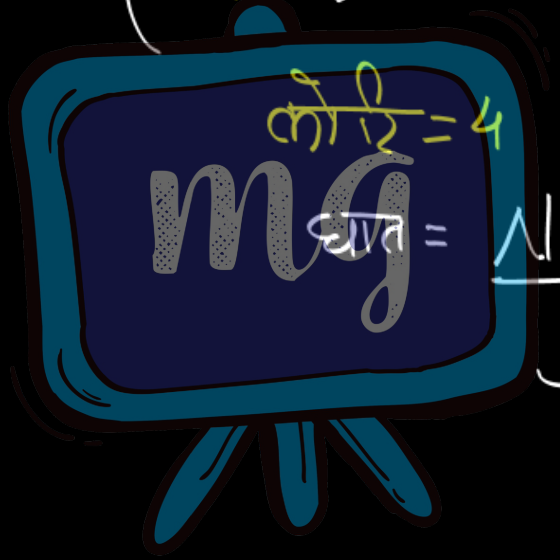


$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + 1 = 0$$



$$\left(\frac{d^4y}{dx^4}\right)' + \sin(y''') = 0$$

कोटि = 4



घात = Not defined

अपरिभाषित

$$\frac{dy}{dx}+1=\sqrt{\frac{d^2y}{dx^2}}$$

दोनों तरफ वर्ग

$$\left(\frac{dy}{dx}+1\right)^2=\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2$$

$$\left(\frac{dy}{dx}+1\right)^2=\frac{d^2y}{dx^2}$$

कोटि = 2

घात = $\left(\frac{1}{2}\right)$ x

= 1

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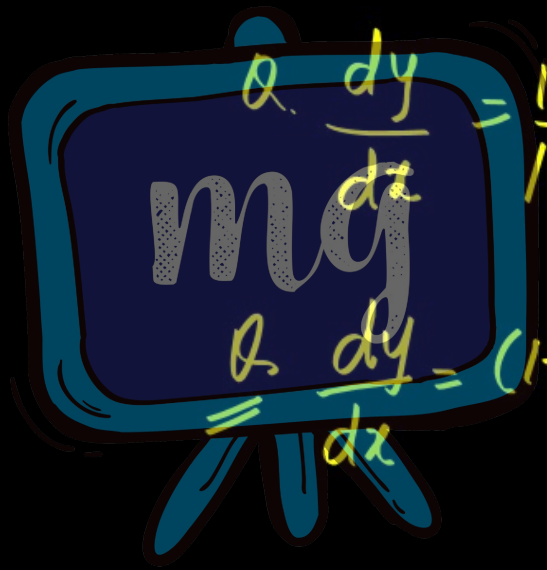
Q. $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^4y}{dx^4}\right)^2 - 5\frac{dy}{dx} = 0$



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चर पृथक्करण विधि



Q. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Q. $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

$\cos x = 1 - 2\sin^2\frac{x}{2}$

$\cos x - 1 = -2\sin^2\frac{x}{2}$

$1 - \cos x = 2\sin^2\frac{x}{2}$

Q. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$dy = \frac{1-\cos x}{1+\cos x}\,dx$

$y = \int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}\,dx$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

$$y = \int \tan^2\frac{x}{2}\,dx$$

$$y = \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$y = \int \sec^2\frac{x}{2}\,dx - \int 1 \cdot dx$$

$$y = \frac{\tan\frac{x}{2}}{\frac{1}{2}} - x + C$$

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$$y = 2\tan\frac{x}{2} - x + C$$



व्यापक हल

$$\left\{\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c\right\}$$

Q. $\dfrac{dy}{dx} = (1+x^2)(1+y^2)$

$$\int \frac{1}{1+y^2}\,dy = \int (1+x^2)\,dx$$

$$\tan^{-1}y = x + \frac{x^3}{3} + C$$

$f(xy) = \frac{x^2+xy}{y^2-xy}$

समघातीय अवकल समीकरण

Q. $(x-y)dx - (x+y)dy = 0$

हल :- $(x-y)dx = (x+y)dy$

$\frac{(x-y)}{x+y} = \frac{dy}{dx}$

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माना $F(x,y) = \frac{x-y}{x+y}$



$$F(\lambda x, \lambda y) = \frac{\lambda x - \lambda y}{\lambda x + \lambda y}$$

$$= \frac{\lambda(x-y)}{\lambda(x+y)}$$

$$F(\lambda x, \lambda y) = F(x,y)$$

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$v = \frac{y}{x}$

$\frac{dy}{dx}$ | $\frac{dx}{dy}$

$y = vx$ | $x = vy$

अवकलन

$$\frac{dy}{dx} = \frac{d}{dx} vx$$

$$\frac{dy}{dx} = v\frac{d}{dx}x + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$



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$$\frac{x-y}{x+y} = \frac{dy}{dx}$$

$$\frac{x - vx}{x + vx} = v + x\frac{dv}{dx}$$

$$\frac{x(1-v)}{x(1+v)} = v + x\frac{dv}{dx}$$

$$\frac{1-v}{1+v} - \frac{v}{1} = x\frac{dv}{dx}$$

$$\frac{1 - v - v(1+v)}{1+v} = x\frac{dv}{dx}$$

$$\frac{1 - v - v - v^2}{1+v} = x\frac{dv}{dx}$$

$$\frac{1 - 2v - v^2}{1+v} = x\frac{dv}{dx}$$

$$\int \frac{1}{x}dx = \int \frac{1+v}{1-2v-v^2}dv$$

$$\int -\frac{1}{x}dx = \int \frac{1+v}{v^2+2v+1}dv$$

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$\{ m\log n = \log n^m \}$

$$-\log x = \frac{1}{2}\int \frac{1}{t}\,dt$$

$$-\log x = \frac{1}{2}\log|t| + \log c$$

$$\log x^{-1} = \log t^{1/2} + \log c$$

$$\log\frac{1}{x} = \log\sqrt{t}\cdot c$$

$$\frac{1}{x} = \sqrt{v^2+2v+1}\cdot C$$

$v^2+2v+1 = t$

$$2v+2 = \frac{dt}{dv}$$

$$2(v+1) = \frac{dt}{dv}$$

$$(v+1)\,dv = \frac{1}{2}\,dt$$

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$$\frac{1}{x^2} = \sqrt{\left(\frac{y}{x}\right)^2 + \frac{2y}{x} + 1} \cdot C$$

$$\frac{1}{x^2} = \sqrt{\frac{y^2 + 2yx + x^2}{x^2}} \cdot C$$

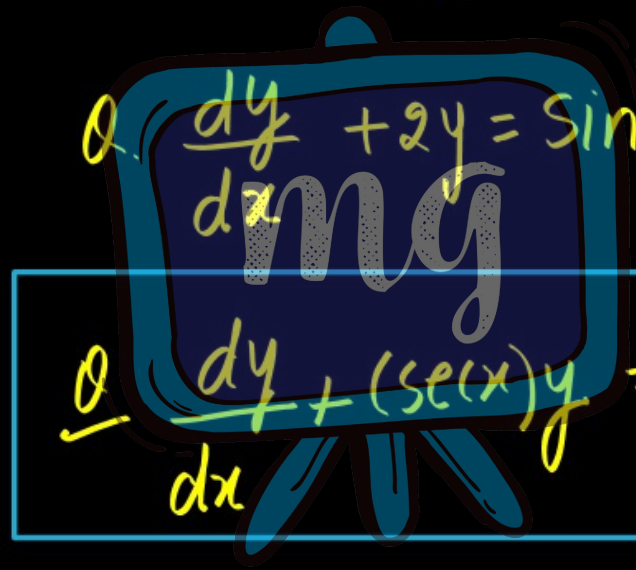


$$\frac{1}{x^2} = \frac{\sqrt{y^2 + 2yx + x^2}}{x}$$

$$\frac{1}{x} = \sqrt{y^2 + 2yx + x^2} \cdot C$$

रैखिक अवकल समीकरण

Q. $\frac{dy}{dx} + 2y = \sin x$



Q $\frac{dy}{dx} + (\sec x)y = \tan x \quad \left[0 \leq x < \frac{\pi}{2}\right]$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Q. $\frac{dy}{dx} + 2y = \sin x$

Soln $P(x) = 2$, $Q(x) = \sin x$

$I.F. = e^{\int P(x)dx}$

$= e^{\int 2\,dx} = e^{2\int 1\,dx} = e^{2x}$

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अवकल समी. का हल –

$$y \cdot I.F. = \int (Q(x) \cdot IF)\, dx + C$$

$$y \cdot e^{2x} = \int \underset{\text{I}}{\sin x} \cdot \underset{\text{II}}{e^{2x}}\, dx + C$$

ILATE

$$= \sin x \int e^{2x}\, dx - \int \left(\frac{d}{dx} \sin x \cdot \int e^{2x}\, dx \right) dx + C$$

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$$y \cdot e^{2x} = \frac{\sin x \cdot e^{2x}}{2} - \int \frac{\cos x \; e^{2x}}{2} dx + C$$

$$y \cdot e^{2x} = \frac{\sin x \cdot e^{2x}}{2} - \frac{1}{2}\int \underset{I}{\cos x} \cdot \underset{II}{e^{2x}} dx + C$$

$$= \frac{\sin x \cdot e^{2x}}{2} - \frac{1}{2}\left[\cos x \int e^{2x} dx - \int \left(\frac{d}{dx}\cos x \cdot \int e^{2x} dx\right) dx\right]$$

$$= \frac{1}{2}\sin x e^{2x} - \frac{1}{2}\left[\frac{\cos x \cdot e^{2x}}{2} + \frac{1}{2}\int \sin x \, e^{2x} dx\right] + C + C$$

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$$y \cdot e^{2x} = \frac{1}{2}\sin x \cdot e^{2x} - \frac{1}{4}\cos x \cdot e^{2x} - \frac{1}{4} y \cdot e^{2x} + C$$

$$\frac{5}{4} y \cdot e^{2x} = \frac{1}{4} e^{2x} [2\sin x - \cos x] + C$$



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$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\frac{dx}{dy} + P(y)x = Q(y)$$

$$I.F. = e^{\int P(x)\,dx}$$

समाकल गुणांक

$$IF = e^{\int P(y)\,dy}$$



Sol:

$$y \cdot I.F = \int Q(x) \times IF\,dx + C$$

$$x \cdot I.F = \int (Q(y) \times IF)\,dy + C$$

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प्रश्न सिद्ध कीजिए $y = e^{-2x}$ अवकल समीकरण $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ का एक हल है।

(2025)



प्रश्न

अवकलन समीकरण $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ की घात है :

अ) 1 ब) 2

स) 3 द) 4

(2024)

प्रश्न तीन कोटि वाले किसी अवकल समीकरण के विशिष्ट हल में उपस्थित स्वेच्छ अचरों की संख्या ..... होती है। [1]

(2024)

प्रश्न सत्यापित कीजिए कि फलन $y = e^x + 1$, अवकल समीकरण $y'' - y' = 0$ का हल है।

(2024)

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प्रश्न अवकल समीकरण $x\frac{dy}{dx}+2y=x^2\left(x\neq 0\right)$ का व्यापक हल ज्ञात कीजिए।

(2024)



प्रश्न अवकल समीकरण $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$ का व्यापक हल ज्ञात कीजिए।

(2024)

प्रश्न y-अक्ष को मूल बिंदु पर स्पर्श करने वाले वृत्तों के कुल का अवकल समीकरण ज्ञात कीजिए।

(2023)

