कक्षा - 12

Chapter - 9

अवकल समीकरण

प्रश्नावली 9.4

(महत्वपूर्ण प्रश्न)

Part - 7

Lakhan Saini

11 से 15 तक के प्रश्नों में प्रत्येक अवकल समीकरण के लिए दिए हुए प्रतिबंध को संतुष्ट करने वाला विशिष्ट हल ज्ञात कीजिए।

11. $(x + y)\, dy + (x - y)\, dx = 0;\ y = 1$ यदि $x = 1$

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हल:- दी गई अवकल समी०-

$$(x+y)\,dy + (x-y)\,dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{(y-x)}{y+x} \quad \text{——— (1)}$$

माना $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - \frac{v}{1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1-v(v+1)}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-(1+v^2)}{v+1}$$

$$\Rightarrow \frac{v+1}{v^2+1}dv = -\frac{1}{x}dx$$ [चर पृथक करने पर]

$$\Rightarrow \int\left[\frac{v}{v^2+1} + \frac{1}{v^2+1}\right]dv = -\int\frac{1}{x}dx$$

[दोनों ओर समाकलन करने पर]

$$\Rightarrow \frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{1}{v^2+1}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{2}\log|v^2+1| + \tan^{-1}v = -\log|x| + c$$

$$\because y = vx \Rightarrow v = \frac{y}{x}$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{y^2}{x^2}+1\right| + \log|x| + \tan^{-1}\left(\frac{y}{x}\right) = c \quad \text{—②}$$

$\because y = 1$ यदि $x = 1$

$\therefore$ समी. ② से $-$

$$\frac{1}{2}\log|1+1| + \log 1 + \tan^{-1}(1) = c$$

$$= \frac{1}{2}\log 2 + 0 + \frac{\pi}{4} = c$$

$$\Rightarrow c = \frac{1}{2}\log 2 + \frac{\pi}{4}$$

अतः समी. (i) का विशिष्ट हल –

$$\frac{1}{2}\log\left|\frac{y^2}{x^2}+1\right| + \log|x| + \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log 2 + \frac{\pi}{4}$$

$$= \log\left|\frac{y^2}{x^2}+1\right| + 2\log|x| + 2\tan^{-1}\left(\frac{y}{x}\right) = \log 2 + \frac{\pi}{2}$$

$$= \log\left|\frac{y^2+x^2}{x^2}\cdot x^2\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = \log 2 + \frac{\pi}{2}$$

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$$= \log(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$$



12. $x^2\,dy + (xy + y^2)\,dx = 0;\ y = 1$

यदि $x = 1$

हल :- दी गई अवकल समी० -

$$x^2 dy + (xy + y^2)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(xy+y^2)}{x^2} \quad \text{—— ①}$$

(समघातीय अवकल समी०)

माना $y = vx$

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} = \frac{-(v+v^2)}{1}$

$\Rightarrow x\frac{dv}{dx} = -(2v+v^2)$

$\Rightarrow \frac{1}{v(2+v)}dv = -\frac{1}{x}dx$ [चर पृथक करने पर]

$\Rightarrow \int\left[\frac{1}{v} - \frac{1}{v+2}\right]dv = -\int\frac{1}{x}dx$

[दोनों ओर समाकलन करने पर]

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$$\Rightarrow \log M - \log|v+2| = -\log|x| + \log c$$

$$\Rightarrow \log\left|\frac{y}{x}\right| - \log\left|\frac{y}{x}+2\right| + \log|x| = \log c \qquad [\because y = vx]$$

$$\Rightarrow \log\left|\frac{y}{y+2x}\right| + \log|x| = \log c$$

$$\Rightarrow \log\left|\frac{xy}{y+2x}\right| = \log c$$

$$\Rightarrow xy = c(y+2x) \quad \text{——} \quad (2)$$

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$\because$ $y=1$ यदि $x=1$

$\therefore$ समी. (2) से -

$$1(1) = C(1+2(1))$$

$$\Rightarrow C = \frac{1}{3}$$

अत: समी (1) का विशिष्ट हल -

$$xy = \frac{1}{3}(y+2x)$$

$$\Rightarrow \boxed{y+2x = 3xy}$$

13. $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0; y=\frac{\pi}{4}$

यदि $x=1$

हल: दी गई अवकल समी० -

$$\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-x\sin^2\left(\frac{y}{x}\right)}{x} \quad \text{—(1)}$$

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माना $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} = v - \sin^2 v$$

$$\Rightarrow x\frac{dv}{dx} = -\sin^2 v$$

$$\Rightarrow -\operatorname{cosec}^2 v\, dv = \frac{1}{x}dx$$

$$\Rightarrow -\int \operatorname{cosec}^2 v\, dv = \int \frac{1}{x}dx$$

$$= \cot v = \log|x| + c$$

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$$\Rightarrow \cot\left(\frac{y}{x}\right) = \log|x| + c \quad \text{—— (2)}$$

$\because y = \frac{\pi}{4}$ यदि $x = 1$

$$\cot\frac{\pi}{4} = \log 1 + c \Rightarrow \boxed{c = 1}$$

अतः सभी (1) का विशिष्ट हल –

$$\cot\left(\frac{y}{x}\right) = \log|x| + 1$$

$$\Rightarrow \cot\left(\frac{y}{x}\right) = \log|x| + \log e$$

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$\Rightarrow \cot\left(\frac{y}{x}\right) = \log|ex|$



14. $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0; y = 0$ यदि $x = 1$

हल :- दी गई अवकल समी० -

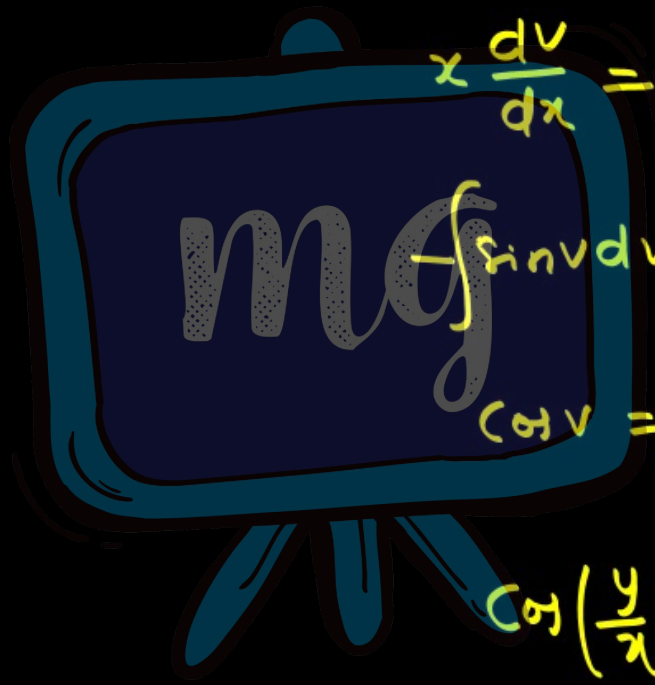
$$\frac{dy}{dx} = \frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right) \quad \text{---} (1)$$

माना $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} = v - \text{cosec}\, v$$

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$$x\frac{dv}{dx} = -\operatorname{cosec} v$$

$$-\int \sin v\, dv = \int \frac{1}{x}\, dx$$

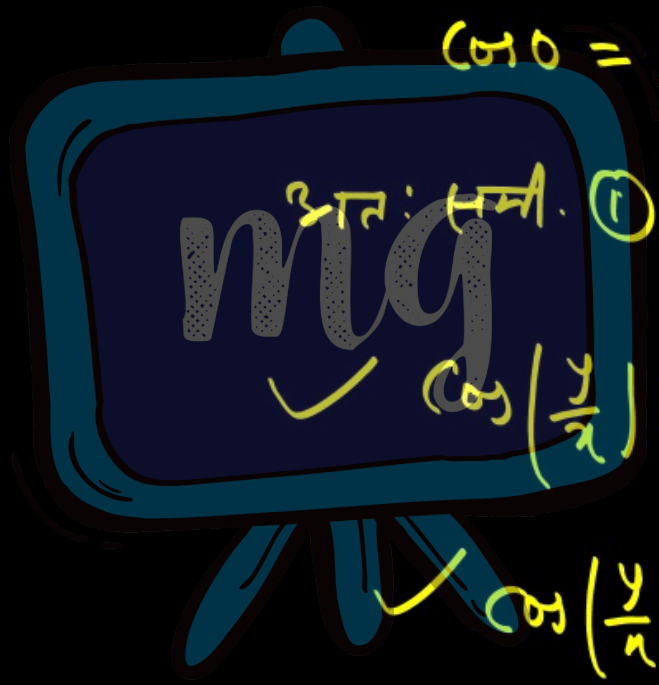
$$\cos v = \log x + c$$

$$\cos\left(\frac{y}{x}\right) = \log x + c \quad \text{—(2)}$$

$\because$ $y = 0$ यदि $x = 1$

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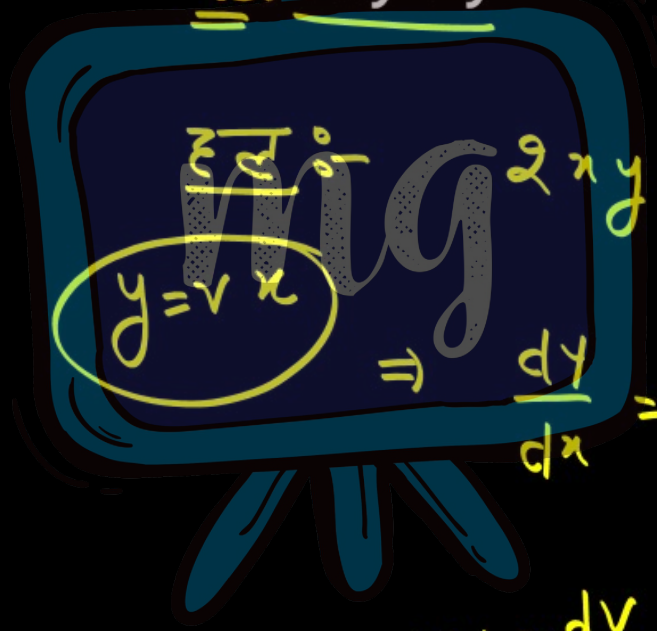
$c_0 = \log 1 + c \Rightarrow c = 1$

अतः सभी (1) का विशिष्ट हल -

$c_1\left(\frac{y}{x}\right) = \log|x| + 1$

$c_2\left(\frac{y}{x}\right) = \log|ex|$

15. $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2$ यदि $x = 1$



हल :-

$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$

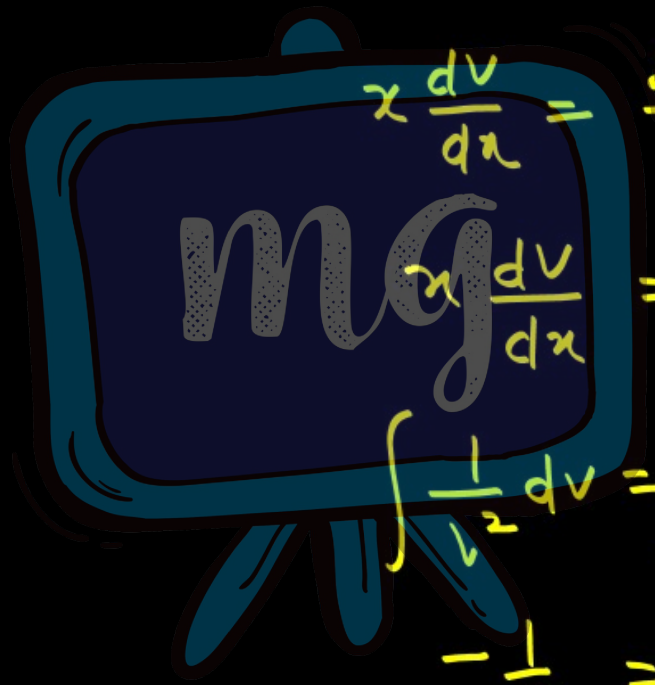
$y = vx$

$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$

$v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$

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$$x\frac{dv}{dx} = \frac{2v+v^2}{2} - v$$

$$x\frac{dv}{dx} = \frac{v^2}{2}$$

$$\int \frac{1}{v^2}\,dv = \int \frac{1}{2x}\,dx$$

$$-\frac{1}{v} = \frac{1}{2}\log|x| + C$$

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$$\Rightarrow -\frac{x}{y} = \frac{1}{2}\log|x| + c \quad \text{——} (2)$$

$y = 2$ यदि $x = 1$

$$-\frac{1}{2} = \frac{1}{2}\log|1| + c$$

$$\boxed{c = -\frac{1}{2}}$$

अतः विशिष्ट हल –

$$-\frac{x}{y} = \frac{1}{2}\log|x| - \frac{1}{2}$$

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$$\frac{x}{y} = \frac{1}{2}(1 - \log|x|)$$

$$y = \frac{2x}{1 - \log|x|}$$

16. $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ के रूप वाले समघातीय अवकल समीकरण को हल करने के लिए निम्नलिखित में से कौन सा प्रतिस्थापन किया जाता है:

(A) $y = vx$ (B) $v = yx$

(C) $x = vy$ (D) $x = v$

17. निम्नलिखित में से कौन सा समघातीय अवकल समीकरण है?

(A) $(4x + 6y + 5)\,dy - (3y + 2x + 4)\,dx = 0$

(B) $(xy)\,dx - (x^3 + y^3)\,dy = 0$

(C) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$

(D) $y^2\,dx + (x^2 - xy - y^2)\,dy = 0$