कक्षा - 12

Chapter - 7

समाकलन

निश्चित समाकलनों के गुणधर्म

प्रश्नावली 7.10

Part - 20

Lakhan Saini

निश्चित समाकलनों के गुणधर्म

उदाहरण-33 :

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ का मान ज्ञात कीजिए।

हल :- $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}\, dx}{\sqrt{\cos x}+\sqrt{\sin x}}$ —— (1)

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$$\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)} + \sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad \text{—— (2)}$$

समी ① + समी ② से –

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$$

$$2I = \int_{\pi/6}^{\pi/3} dx$$

$$2I = [x]_{\pi/6}^{\pi/3}$$

$$2I = \pi/3 - \pi/6$$

$$2I = \pi/6 \Rightarrow \boxed{I = \pi/12}$$

उदाहरण-34 : ****

$\int_0^{\frac{\pi}{2}} \log \sin x \, dx$ का मान ज्ञात कीजिए।

हल :- माना $I = \int_0^{\pi/2} \log \sin x \, dx$ —— (1)

$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$ $\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right.$

$I = \int_0^{\pi/2} \log \cos x \, dx$ —— (2)

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समी. ① + समी. ② से –

$$2I = \int_0^{\pi/2} [\log \sin x + \log \cos x]\, dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \log \frac{2 \sin x \cos x}{2}\, dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} (\log \sin 2x - \log 2)\, dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2\, dx \quad \text{—③}$$

माना $I_1 = \int_0^{\pi/2} \log \sin 2x \, dx$

यदि $2x = t$

$2\,dx = dt$

$dx = \frac{1}{2} dt$

जब $x = 0 \Rightarrow t = 0$

तथा जब $x = \pi/2$

$t = 2(\pi/2)$

$\therefore I_1 = \int_0^{\pi/2} \log \sin 2x \, dx = \frac{1}{2} \int_0^{2(\pi/2)} \log \sin t \, dt$

$\int_0^{2a} f(x)dx = 2 \int_0^{a} f(x)dx$

जब $f(x) = f(2a-x)$

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$$I_1 = \frac{1}{2} \int_0^{\pi/2} \log \sin t \, dt$$

$$I_1 = \int_0^{\pi/2} \log \sin x \, dx$$

$$\left[\int_a^b f(x)\,dx = \int_a^b f(t)\,dt \right]$$

$$I_1 = I$$

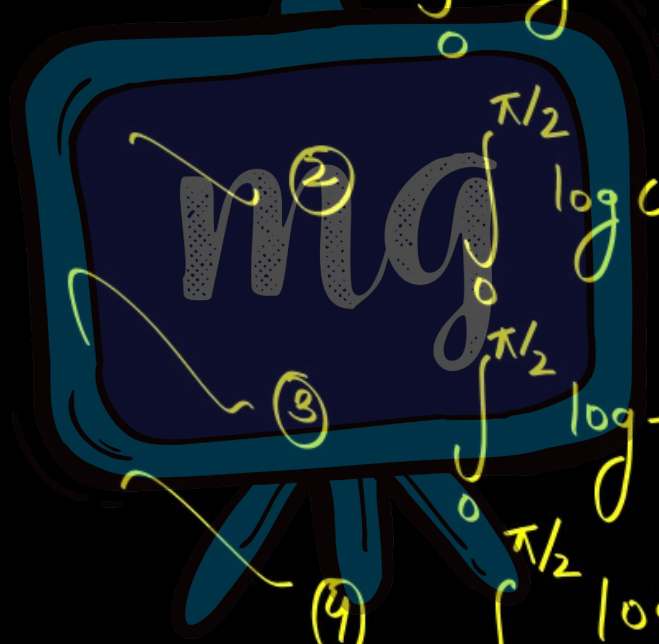
$\therefore$ समी. (3) से –

$$2I = I - \log 2 \int_0^{\pi/2} dx$$

$$I = -\log 2 \left[x \right]_0^{\pi/2} \Rightarrow I = -\frac{\pi}{2} \log 2$$

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① $\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2$

② $\int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

③ $\int_0^{\pi/2} \log \tan x \, dx = 0$

④ $\int_0^{\pi/2} \log \cot x \, dx = 0$

निश्चित समाकलनों के गुणधर्मों का उपयोग करते हुए 1 से 19 तक के प्रश्नों में समाकलनों का मान ज्ञात कीजिए।

1. $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Sol:- $I = \int_0^{\pi/2} \cos^2 x \, dx$ — (1)

$I = \int_0^{\pi/2} \left[\cos\left(\frac{\pi}{2} - x\right)\right]^2 dx$

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$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \sin^2 x\,dx \quad \text{—②}$$

समी ① + समी ② से —

$$2I = \int_0^{\pi/2} (\cos^2 x + \sin^2 x)\,dx$$

$$= [x]_0^{\pi/2} = \pi/2$$

$$\boxed{I = \pi/4}$$

2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

Sol:- $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ — (1)

$\because \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ — (2)

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$$2I = \int_0^{\pi/2} dx$$

$$= [x]_0^{\pi/2}$$

$$2I = \pi/2$$

$$\boxed{I = \pi/4}$$

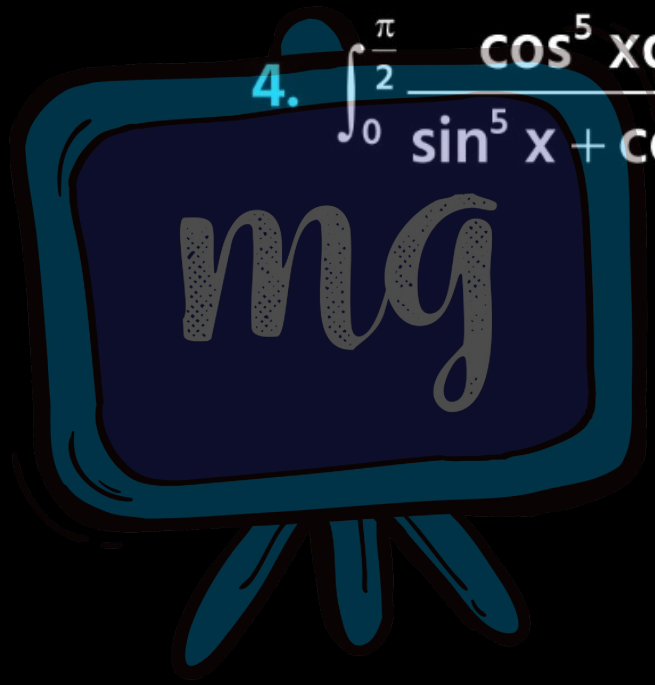
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3. $$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x \, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

$$I = \int_0^{\pi/2} dx$$

$$I = \pi/4$$

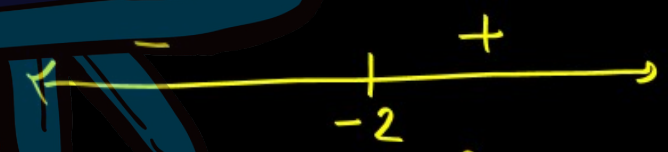
4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$



5. $\int_{-5}^{5} |x+2|\,dx$

हल :

$$|x+2| = \begin{cases} -(x+2) & x \le -2 \\ x+2 & x > -2 \end{cases}$$



$$\therefore \int_{-5}^{5} |x+2|\,dx = -\int_{-5}^{-2} (x+2)\,dx + \int_{-2}^{5} (x+2)\,dx$$

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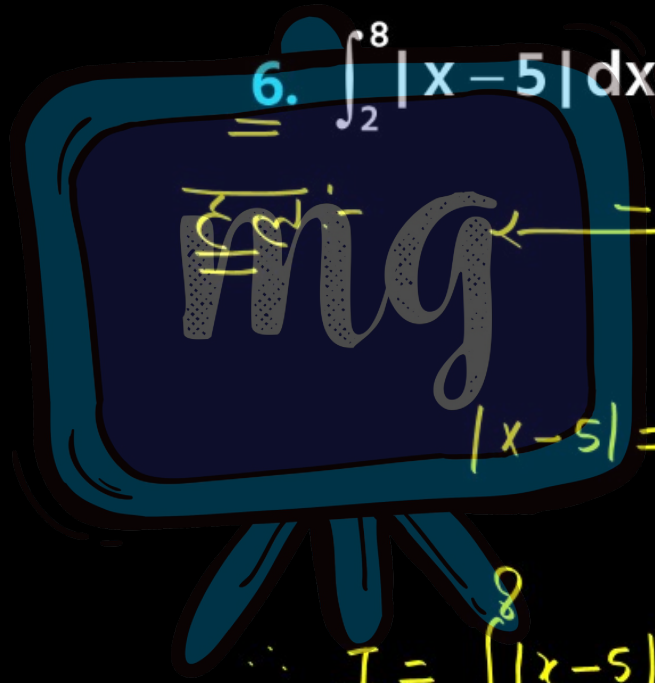
$$= -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[(2-4) - \left(\frac{25}{2} - 10\right)\right] + \left[\left(\frac{25}{2} + 10\right) - (2-4)\right]$$

$$= 2 + \frac{25}{2} - 10 + \frac{25}{2} + 10 + 2$$

$$= 2 + \frac{25}{2} \times 2 + 2 = 29$$

6. $\int_2^8 |x-5|\,dx$



$$|x-5| = \begin{cases} -(x-5) & x \le 5 \\ x-5 & x > 5 \end{cases}$$

$$\therefore I = \int_2^8 |x-5|\,dx = \int_2^5 -(x-5)\,dx + \int_5^8 (x-5)\,dx$$

7. $\int_0^1 x(1-x)^n dx$

माना $I = \int_0^1 x(1-x)^n dx$

$\because \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$

$= \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1}) dx$

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$$I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$I = \frac{1}{n+1} - \frac{1}{n+2}$$

$$I = \frac{(n+2)-(n+1)}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$$

8. $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

हल :- $I = \int_0^{\pi/4} \log\left[1+\tan x\right] dx$ — (1)

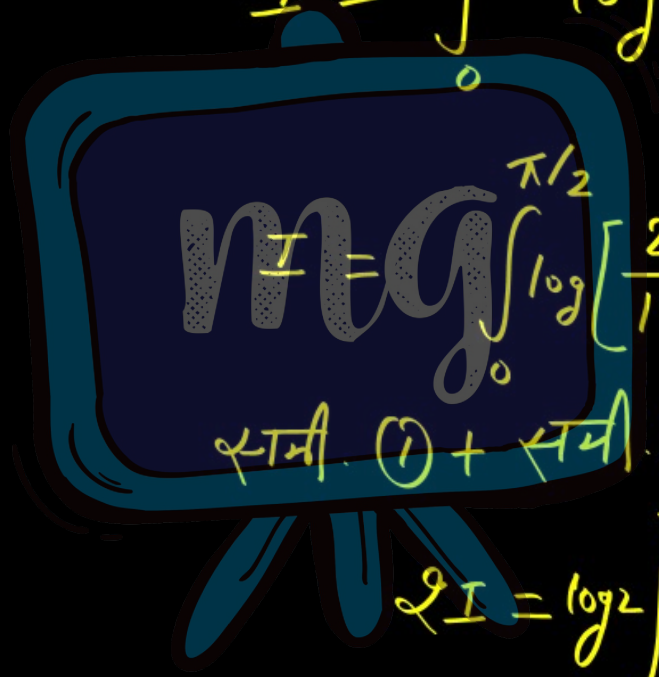
$\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\therefore I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right] dx$

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$$I = \int_0^{\pi/4} \log\left[1 + \frac{\tan\pi/4 - \tan x}{1 + \tan\pi/4 \tan x}\right] dx$$



$$I = \int_0^{\pi/2} \log\left[\frac{2}{1+\tan x}\right] dx \quad \text{—②}$$

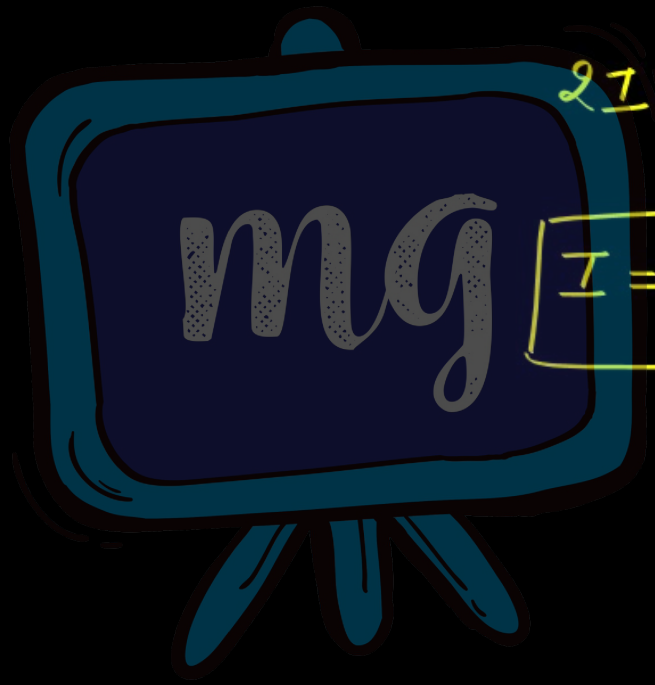
दोनों ① + दोनों ②

$$2I = \log 2 \int_0^{\pi/4} dx$$

$$2I = \log 2 \left[x\right]_0^{\pi/4}$$

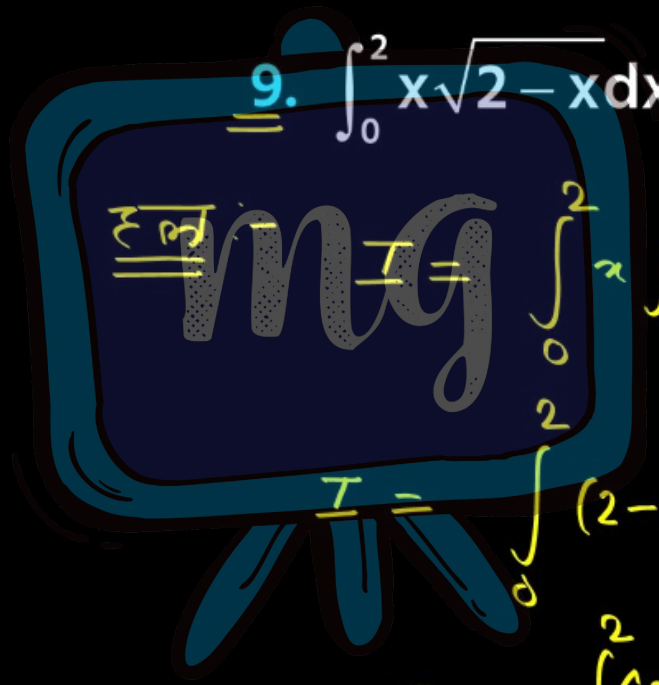
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$$2I = \frac{\pi}{4}\log 2$$

$$\boxed{I = \frac{\pi}{8}\log 2}$$



9. $\int_0^2 x\sqrt{2-x}\,dx$

हल :- $I = \int_0^2 x\sqrt{2-x}\,dx$

$I = \int_0^2 (2-x)\sqrt{x}\,dx$

$I = \int_0^2 (2x^{1/2} - x^{3/2})\,dx$

10. $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

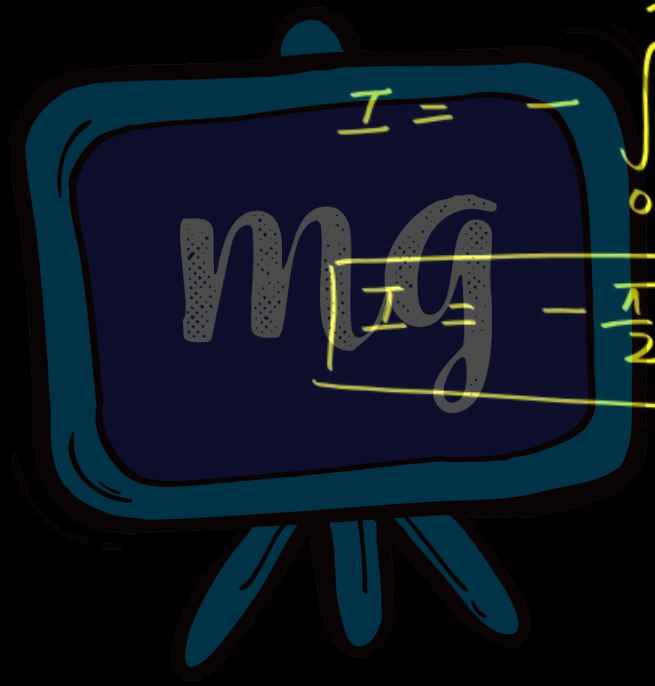
हल :- $I = \int_0^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx$

$I = \int_0^{\pi/2} \log\left[\frac{\sin^2 x}{2\sin x\cos x}\right]dx$

$I = \int_0^{\pi/2} [\log\tan x - \log 2]\,dx$

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$$I = \int_0^{\pi/2} \log 2 \, dx \qquad \left\{ \because \int_0^{\pi/2} \log \tan x = 0 \right\}$$

$$\boxed{I = -\frac{\pi}{2} \log 2}$$

11. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$

$f(-x) = f(x)$

$I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

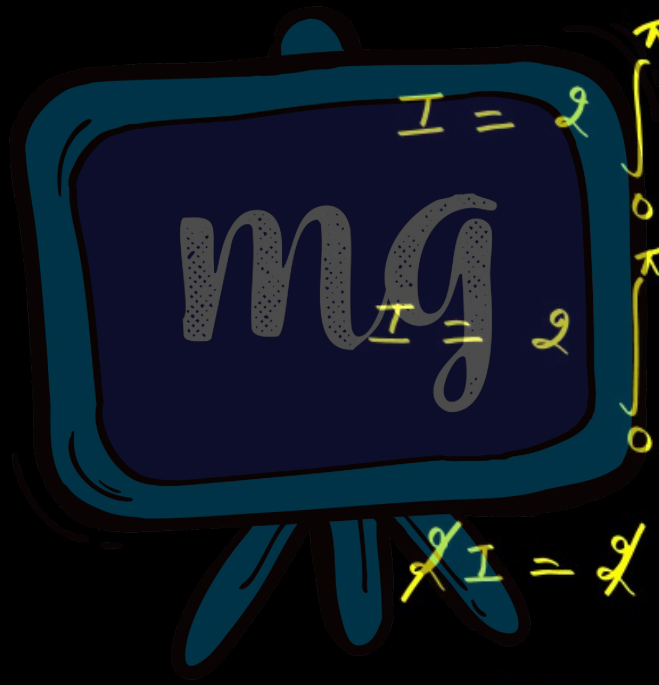
$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

यदि f समकलन हो

$\because [\sin(-x)]^2 = \sin^2 x$

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$$I = 2\int_0^{\pi/2} \sin^2 x\, dx \quad \text{—— (1)}$$

$$I = 2\int_0^{\pi/2} \cos^2 x\, dx \quad \text{—— (2)}$$

$$2I = 2\int_0^{\pi/2} dx$$

$$\boxed{I = \pi/2}$$