कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 9

प्रश्नावली 5.3

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6. $x^3 + x^2y + xy^2 + y^3 = 81.$

$$\frac{d}{dx}(x^3+x^2y+xy^2+y^3) = \frac{d}{dx}(81)$$

$$\Rightarrow \frac{d}{dx}(x^3)+\frac{d}{dx}(x^2y)+\frac{d}{dx}(xy^2)+\frac{d}{dx}y^3 = \frac{d}{dx}(81)$$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + y(2x) + x\left(2y\frac{dy}{dx}\right) + y^2 + 3y^2\frac{dy}{dx} = 0$$

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$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 + y^2 + 2xy + (x^2 + 3y^2 + 2xy)\frac{dy}{dx} = 0$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-(3x^2 + y^2 + 2xy)}{x^2 + 3y^2 + 2xy}}$$

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7. $\sin^2 y + \cos xy = \kappa$.

$$\frac{d}{dx}(\sin^2 y) + \frac{d}{dx}\cos(xy) = \frac{d}{dx}\kappa$$

$$\Rightarrow 2\sin y \frac{d}{dx}\sin y - \sin(xy)\frac{d}{dx}(xy) = 0$$

$$\Rightarrow 2\sin y \cdot \cos y \frac{dy}{dx} - \sin xy\left[x\frac{dy}{dx} + y\right] = 0$$

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$$\Rightarrow \sin^2 y \frac{dy}{dx} - x \sin^2 xy \frac{dy}{dx} - y \sin^2 xy = 0$$

$$\Rightarrow (\sin^2 y - x \sin^2 xy) \frac{dy}{dx} = y \sin^2 xy$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{y \sin^2 xy}{\sin^2 y - x \sin^2 xy}}$$

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8. $\sin^2 x + \cos^2 y = 1.$

$$\frac{d}{dx}(\sin^2 x + \cos^2 y) = \frac{d}{dx}(1)$$

$$\Rightarrow \frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = 0$$

$$\Rightarrow 2\sin x \frac{d}{dx}\sin x + 2\cos y \frac{d}{dx}\cos y = 0$$

$$\Rightarrow 2\sin x \cdot \cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$$

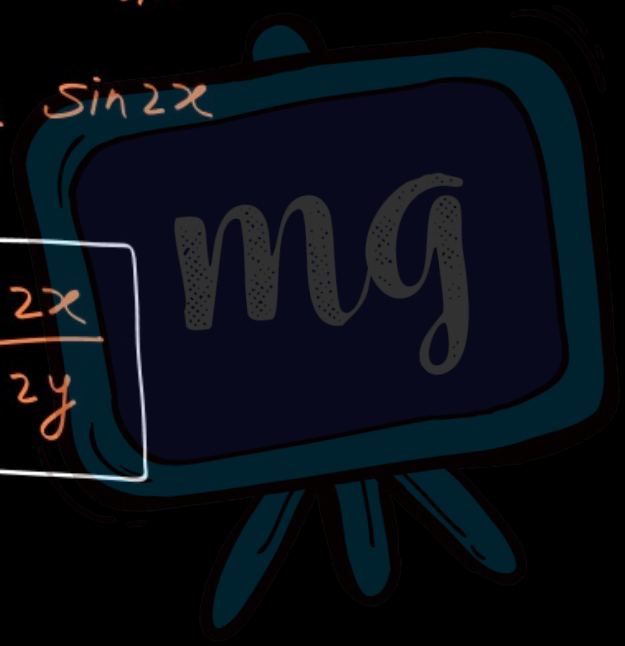
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$$\Rightarrow \sin 2x - \sin 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \sin 2y \frac{dy}{dx} = \sin 2x$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}}$$



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9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

माना $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(\sin 2\theta)$

$\Rightarrow y = 2\theta$

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अब, $y = 2\tan^{-1}x$

यहाँ $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$$\Rightarrow \frac{dy}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = 2\cdot\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{1+x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

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10. $y = \tan^{-1}\left(\frac{3x - x^3}{1 + 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

माना $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 + 3\tan^2\theta}\right)$

$\Rightarrow y = \tan^{-1}.\tan 3\theta$

$\Rightarrow y = 3\theta$

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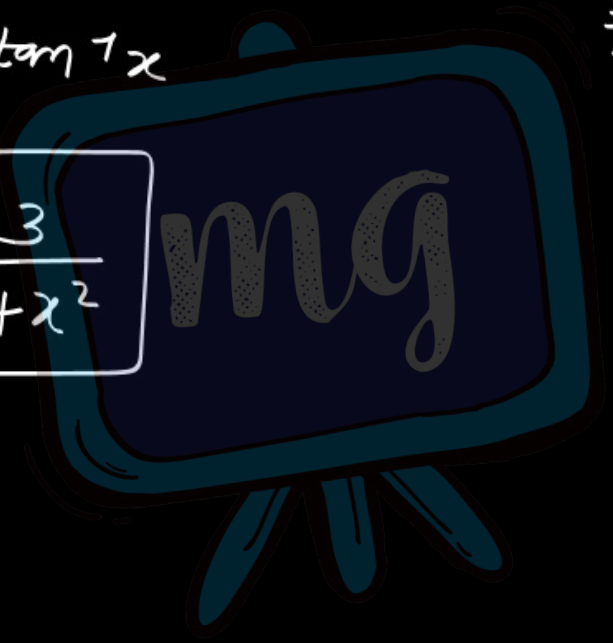
अब, $y = 3\theta$

जहाँ $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$y = 3\tan^{-1}x$

$$\boxed{\frac{dy}{dx} = \frac{3}{1+x^2}}$$



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11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

माना $x = \tan\theta$

$\Rightarrow y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}\cdot\cos 2\theta$

$\Rightarrow y = 2\theta$

अब, $y = 2\theta$

यहाँ $x = \tan\theta$

$\Rightarrow y = 2\tan^{-1}x$

$\Rightarrow \theta = \tan^{-1}x$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{1+x^2}}$$

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12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

माना $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}\cos 2\theta$

$\Rightarrow y = \sin^{-1}(\cos 2\theta)$

$\Rightarrow y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$

$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x$

$\Rightarrow \frac{dy}{dx} = 0 - \frac{2}{1+x^2}$

यहाँ $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$$\boxed{\frac{dy}{dx} = \frac{-2}{1+x^2}}$$

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13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

माना $x = \tan\theta$

$\Rightarrow y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}\sin 2\theta$

$\Rightarrow y = \cos^{-1}\cdot\cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$

$\Rightarrow y = \frac{\pi}{2} - 2 \cdot \tan^{-1} x$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-2}{1+x^2}}$

यहाँ $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

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14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

माना $x = \sin\theta$

$\Rightarrow y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\Rightarrow y = \sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(2\sin\theta\cdot\cos\theta)$

$\Rightarrow y = \sin^{-1}\sin 2\theta$

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$y = 2\theta$

$\Rightarrow y = 2\sin^{-1}x$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}}$

जहाँ $x = \sin\theta$

$\Rightarrow \theta = \sin^{-1}x$

$$\boxed{\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}}$$

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15. $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), 0 < x < \frac{1}{\sqrt{2}}$

माना $x = \cos\theta$

$\Rightarrow y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

$\Rightarrow y = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow y = \sec^{-1}\sec 2\theta$

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$y = 2\theta$

$y = 2\cos^{-1}x$

यहाँ $x = \cos\theta$

$\Rightarrow \theta = \cos^{-1}x$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$