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कक्षा-10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

(Introduction To Trigonometry)

भाग - 6

प्रश्नावली 8.3

By : Rajendra Yadav Sir

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त्रिकोणमितीय सर्वसमिकाएँ

उदाहरण 12 - सर्वसमिका $\sec^2\theta = 1 + \tan^2\theta$ का प्रयोग करके सिद्ध कीजिए कि

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$

LHS: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

अंश व हर में $\cos\theta$ का भाग देने पर:

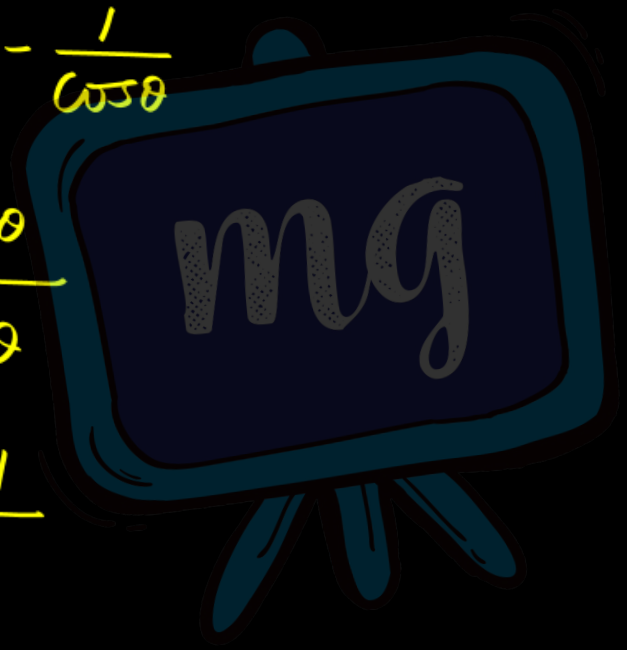
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$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$\Rightarrow \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$\Rightarrow \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$



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अंश व हर को $(\tan\theta - \sec\theta)$ से गुणा करने पर:

$$\frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \times \frac{\tan\theta - \sec\theta}{(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

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अब, $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow \tan^2\theta - \sec^2\theta = -1$

अत: $$\frac{-1-(\tan\theta-\sec\theta)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$$

$$= \frac{-1-\tan\theta+\sec\theta}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$$

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$$= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

RHS

इति सिद्धम्।

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1. त्रिकोणमितीय अनुपातों sin A, sec A और tan A को cot A के पदों में व्यक्त कीजिए।

1) $\sin A \longrightarrow \cot A$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$

$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$

$\Rightarrow \boxed{\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}}$

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2) $\sec A \longrightarrow \cot A$

$$\sec A = \frac{1}{\cos A}$$

$$= \frac{1}{\sqrt{1-\sin^2 A}}$$

$$= \frac{1}{\sqrt{1-\frac{1}{1+\cot^2 A}}}$$

$$\sec A = \frac{1}{\sqrt{\frac{1+\cot^2 A - 1}{1+\cot^2 A}}}$$

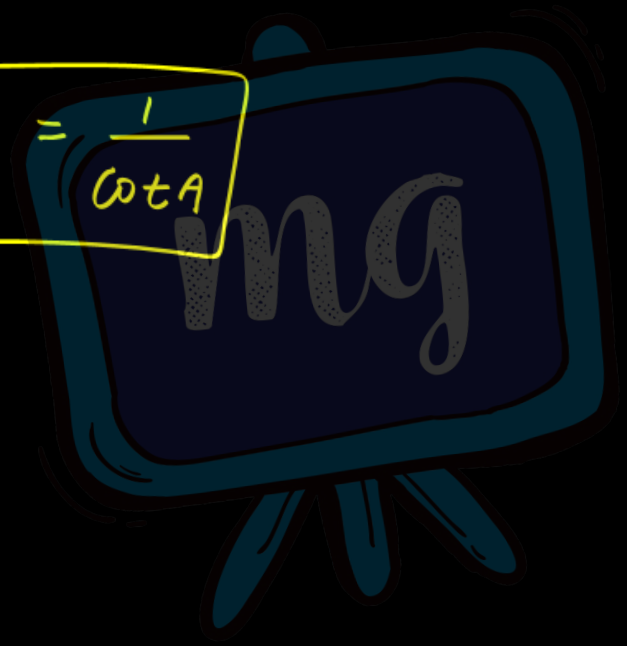
$$\sec A = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

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3) $\tan A \longrightarrow \cot A$

$$\tan A = \frac{1}{\cot A}$$



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1) $\sin A \longrightarrow \sec A$

$\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$

$\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$

$\sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$

$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

2. ∠A के अन्य सभी त्रिकोणमितीय अनुपातों को sec A के पदों में लिखिए।

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2) $\cos A \longrightarrow \sec A$

$$\cos A = \frac{1}{\sec A}$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

3) $\tan A \longrightarrow \sec A$

$$1 + \tan^2 A = \sec^2 A$$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

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4) $\cot A \longrightarrow \sec A$

$$\cot A = \frac{1}{\tan A}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

5) $\operatorname{cosec} A \longrightarrow \sec A$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$= \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

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$9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1$

$= 9$

3. सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए :

(i) $9\sec^2 A - 9\tan^2 A$ बराबर है:

(A) 1 (B) 9

(C) 8 (D) 0

$1 + \tan^2 A = \sec^2 A$

$\Rightarrow \sec^2 A - \tan^2 A = 1$

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(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ बराबर है:

(A) 0 (B) 1

(C) 2 (D) –1

$$\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\times\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cdot\cos\theta}=\frac{(\sin^2\theta+\cos^2\theta)+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}$$

$$=\frac{2\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}=2$$

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$$\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

(iii) (sec A + tan A) (1 – sin A) बराबर है:

(A) sec A (B) sin A

(C) cosec A (D) cos A

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$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1/\cos^2 A}{1/\sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$ बराबर है:

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

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LHS: $(\operatorname{cosec}\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

4. निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

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$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \qquad (\because \sin^2\theta = 1-\cos^2\theta)$$

$$= \frac{(1-\cos\theta)^{\cancel{2}}}{(1+\cos\theta)(1-\cancel{\cos\theta})}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

RHS

इति सिद्धम्।