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कक्षा – 12

गणित

प्रतिलोम त्रिकोणमितीय फलन

अध्याय – 2

भाग - 3

प्रश्नावली 2.2

By : Rajendra Yadav Sir

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उदाहरण - 3

दर्शाइए कि

(i) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x, -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$

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(11) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$

माना $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

LHS : $\sin^{-1}(2x\sqrt{1-x^2})$

$\Rightarrow \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\Rightarrow \sin^{-1}(2\sin\theta\sqrt{\cos^2\theta})$

$\Rightarrow \sin^{-1}(2\sin\theta\cdot\cos\theta)$

$\Rightarrow \sin^{-1}(\sin 2\theta)$

$\because 2\sin\theta\cdot\cos\theta = \sin 2\theta$

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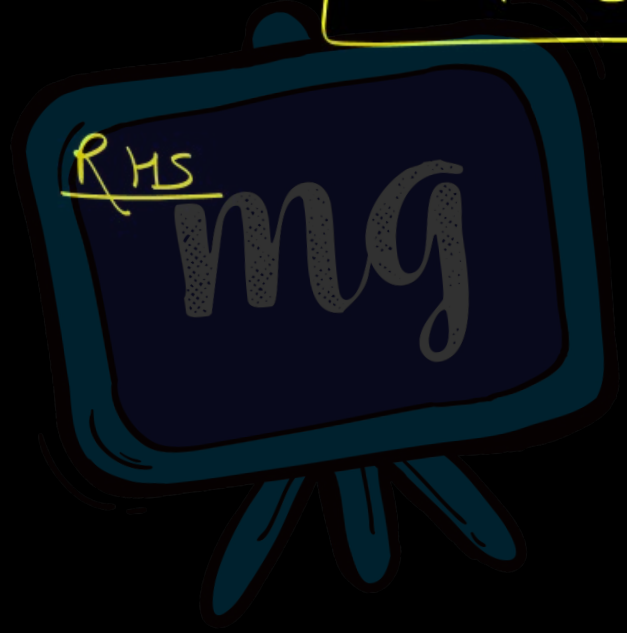
$= \sin^{-1}.\sin 2\theta$

$= 2\theta$

$= 2\sin^{-1} x$

$\because \sin^{-1}.\sin x = x$

RHS



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(ii) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x$

माना $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

LHS: $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$\Rightarrow \sin^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$

$\Rightarrow \sin^{-1}\left(2\cos\theta\sqrt{\sin^2\theta}\right)$

$\Rightarrow \sin^{-1}\left(2\cos\theta\cdot\sin\theta\right)$

$\Rightarrow \sin^{-1}\left(2\sin\theta\cdot\cos\theta\right)$

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$\Rightarrow \sin^{-1}(\sin 2\theta)$

$\because 2\sin\theta \cdot \cos\theta = \sin 2\theta$

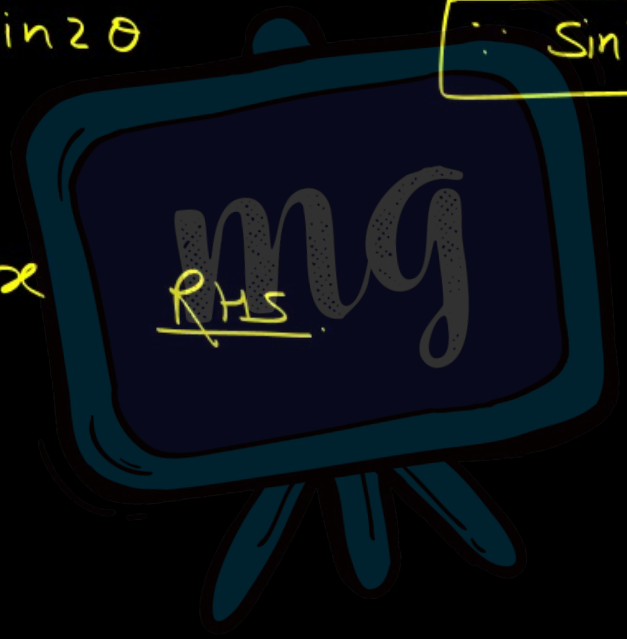
$\Rightarrow \sin^{-1} \cdot \sin 2\theta$

$\because \sin^{-1} \cdot \sin x = x$

$\Rightarrow 2\theta$

$\Rightarrow 2\cos^{-1} x$

RHS.



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उदाहरण - 4

$\tan^{-1}\frac{\cos x}{1-\sin x}, \frac{-3\pi}{2} < x < \frac{\pi}{2}$ को सरलतम रूप में व्यक्त कीजिए।

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$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$$

$$\Rightarrow \tan^{-1}\left[\frac{\cos^2 x/2 - \sin^2 x/2}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right]$$

$\because \cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cdot\cos\theta$

$a^2 - b^2 = (a+b)(a-b)$

$(a-b)^2 = a^2 + b^2 - 2ab$

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$$\Rightarrow \tan^{-1}\left[\frac{1+\frac{\sin x/2}{\cos x/2}}{1-\frac{\sin x/2}{\cos x/2}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{1+\tan x/2}{1-\tan x/2}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right]$$

$$\Rightarrow \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}$$

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$$\Rightarrow \tan^{-1}\cdot \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$\Rightarrow \frac{\pi}{4}+\frac{x}{2}$$



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उदाहरण - 5

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x>1$ को सरलतम रूप में लिखिए।

माना $x = \sec\theta \Rightarrow \theta = \sec^{-1} x$

LHS: $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right)$

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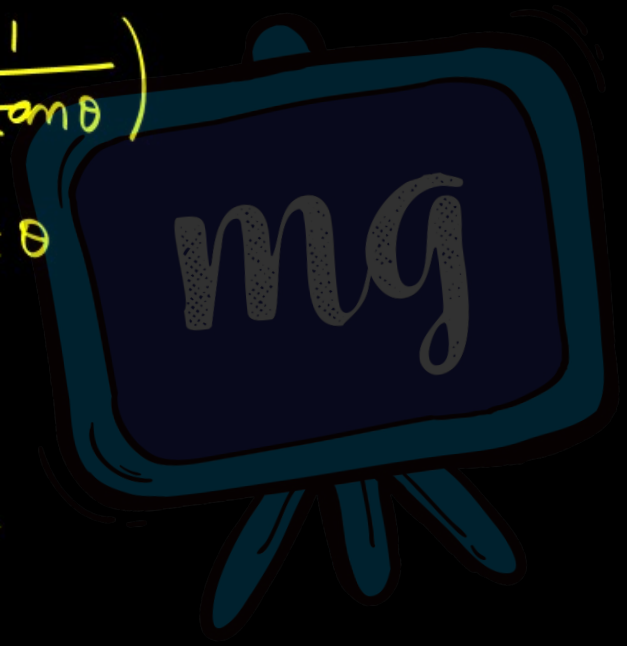
$$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{\tan^2\theta}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\tan\theta}\right)$$

$$\Rightarrow \cot^{-1}.\cot\theta$$

$$\Rightarrow \theta$$

$$\Rightarrow \sec^{-1}x.$$



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प्रश्नावली 2.2

निम्नलिखित को सिद्ध कीजिए:

Q1. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

माना $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

RHS: $\sin^{-1}(3x - 4x^3)$

$\Rightarrow \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\Rightarrow \sin^{-1}(\sin 3\theta)$

$\Rightarrow \sin^{-1}.\sin 3\theta$

$\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

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$\Rightarrow 3\theta$

$\Rightarrow 3\sin^{-1}x$



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प्रश्नावली 2.2

Q2. $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$

माना $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

RHS: $\cos^{-1}(4x^3 - 3x)$

$\Rightarrow \cos^{-1}[4\cos^3\theta - 3\cos\theta]$

$\Rightarrow \cos^{-1}(\cos 3\theta)$

$\Rightarrow 3\theta$

$\because 4\cos^3\theta - 3\cos\theta = \cos 3\theta$

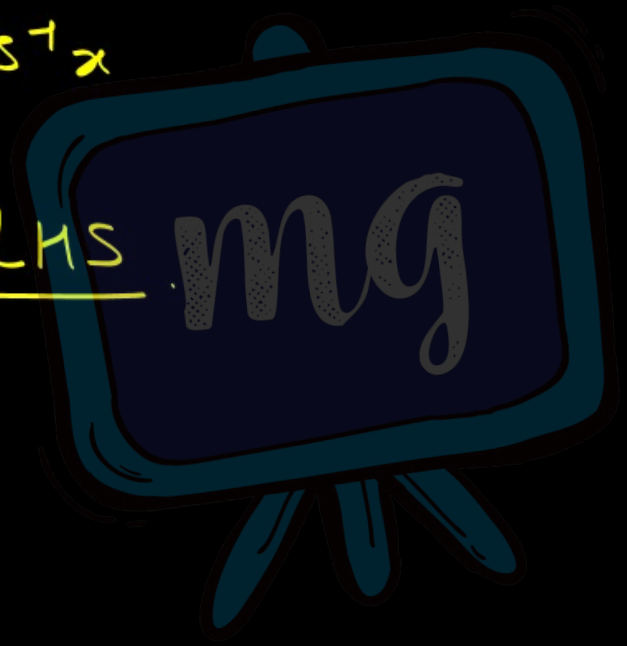
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$\Rightarrow \quad 3\theta$

$\Rightarrow \quad 3\cos^{-1}x$

LHS



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प्रश्नावली 2.2

निम्नलिखित फलनों को सरलतम रूप में लिखिए:

Q3. $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

माना $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

अत: $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

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$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{\sec^2\theta - 1}}{\tan\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{\cos\theta} - 1}{\sin\theta / \cos\theta}\right)$$

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$$\Rightarrow \tan^{-1}\left[\frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right]$$

$\because \cos 2\theta = 1 - 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta = 1 - \cos 2\theta$

$\because \sin 2\theta = 2\sin\theta \cdot \cos\theta$

$$\Rightarrow \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2 \cdot \cos\theta/2}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta/2}{\cos\theta/2}\right)$$

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$$\Rightarrow \tan^{-1} \tan \theta/2$$

$$\Rightarrow \frac{\theta}{2}$$

$$\Rightarrow \frac{1}{2} \cdot \tan^{-1} x$$

