

कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 5

केशव शर्मा



① $\frac{1}{7\sqrt{5}}$

$$\frac{1}{7\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$\frac{\sqrt{5}}{7 \times 5} = \frac{\sqrt{5}}{35}$$

$$\textcircled{2} \frac{2}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$(a-b) \quad (a+b) = a^2 - b^2$

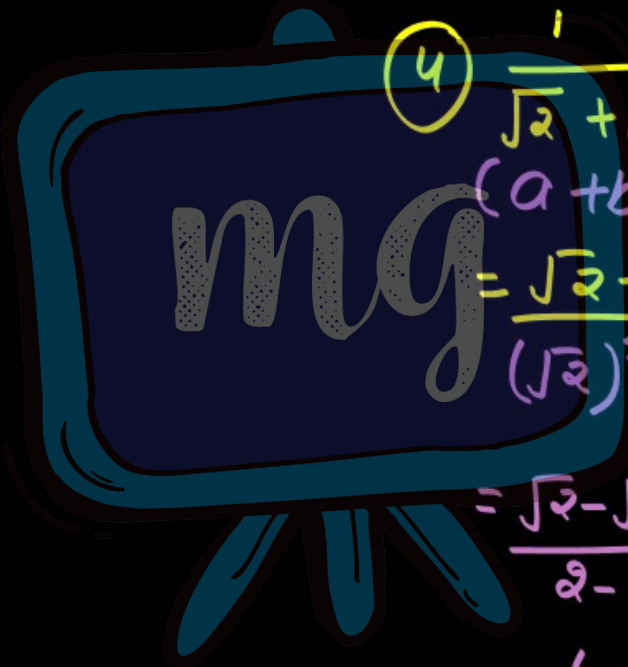
$$\frac{2(2+\sqrt{5})}{2^2 - (\sqrt{5})^2} = \frac{4+2\sqrt{5}}{4-5}$$

$$= \frac{4+2\sqrt{5}}{-1}$$

$$= -(4+2\sqrt{5})$$

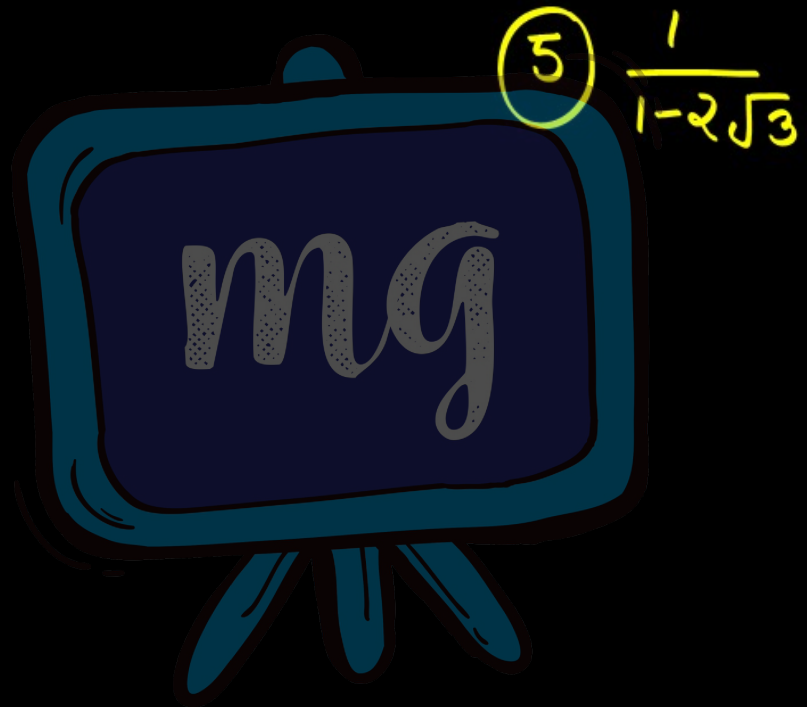
$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned} \textcircled{3} \quad & \frac{\sqrt{3}+2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2} \\ &= \frac{(\sqrt{3}+2)^2}{(\sqrt{3})^2 - 2^2} \\ &= \frac{(\sqrt{3})^2 + 2^2 + 2 \times \sqrt{3} \times 2}{3 - 4} \\ &= \frac{3 + 4 + 4\sqrt{3}}{-1} \\ &= -(7 + 4\sqrt{3}) \end{aligned}$$



(4) $\frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}}$

$$\begin{aligned} & (a+b)(a-b) = a^2 - b^2 \\ & = \frac{\sqrt{2} - \sqrt{3}}{(\sqrt{2})^2 - (\sqrt{3})^2} \\ & = \frac{\sqrt{2} - \sqrt{3}}{2 - 3} = \frac{\sqrt{2} - \sqrt{3}}{-1} \\ & = -(\sqrt{2} - \sqrt{3}) \end{aligned}$$



प्रश्नावली 8.2

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

$$\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}} \quad \tan 45^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} \quad \cot 45^\circ = 1$$

Q.1 निम्नलिखित के मान ज्ञात कीजिए

(iv)
$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\begin{aligned} & \frac{3\sqrt{3} - 4}{2\sqrt{3}} \\ &= \frac{4 + 3\sqrt{3}}{2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{हल:- } & \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} \\ &= \end{aligned}$$

$$= \frac{(3\sqrt{3})^2 + 4^2 - 2 \times 3\sqrt{3} \times 4}{9 \times 3 - 16}$$

$$= \frac{9 \times 3 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{11}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

परिमेय करण

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

(a+b) (a-b) = a² - b²

$$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$\underline{\underline{9 \times 3}}$$

$$\left\{ \begin{array}{l} \cos 60^\circ = \frac{1}{2} \\ \cos 30^\circ = \frac{\sqrt{3}}{2} \\ \sec 30^\circ = \frac{2}{\sqrt{3}} \\ \sin 30^\circ = \frac{1}{2} \end{array} \right.$$



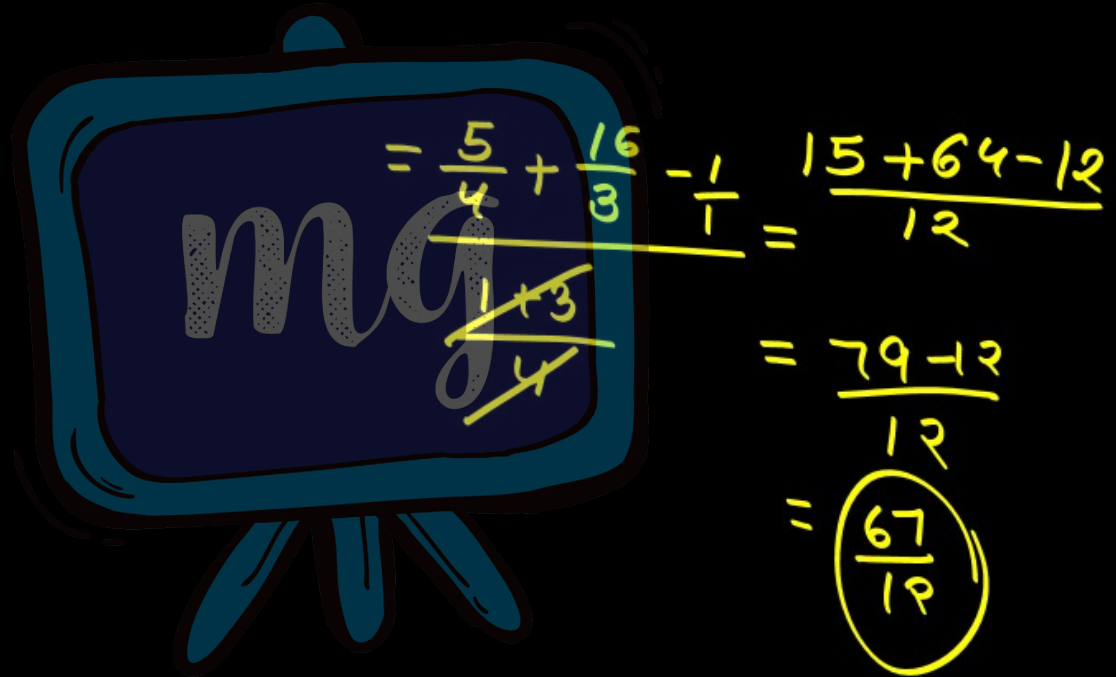
(v)

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

हल:-

$$\frac{5 \cdot \left(\frac{1}{2}\right)^2 + 4 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 4 \cdot \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$


$$\begin{aligned} &= \frac{5}{4} + \frac{16}{3} - \frac{1}{1} = \frac{15 + 64 - 12}{12} \\ &= \frac{79 - 12}{12} \\ &= \frac{67}{12} \end{aligned}$$

$$\frac{2 + \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$2 \left(\frac{1}{\sqrt{3}} \right)$$

$$\frac{1 + \left(\frac{1}{\sqrt{3}} \right)^2}{1 + \left(\frac{1}{\sqrt{3}} \right)^2}$$



Q.2 सही विकल्प चुनिए और अपने विकल्प का औचित्य दीजिए :

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

(अ) $\sin 60^\circ \frac{\sqrt{3}}{2}$

(स) $\tan 60^\circ$

(ब) $\cos 60^\circ$

(द) $\sin 30^\circ$

$$\begin{aligned}
 &= \frac{8}{2 \times 2} \times \sqrt{3} \\
 &= \frac{\sqrt{3}}{2} \\
 &= \frac{\frac{2}{\sqrt{3}}}{3+1} = \frac{\frac{2}{\sqrt{3}}}{4} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}} \\
 &= \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2 \times \sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{6}
 \end{aligned}$$

Aim 100



(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - (1)^2}{1 + 1} = \frac{1 - 1}{2} = \frac{0}{2} = 0$

(अ) $\tan 90^\circ$
(ब) $\sin 45^\circ$

(ब) 1
(अ) 0

$$A = 45^\circ$$

$$\begin{aligned}\sin 2A &= \sin 2 \cdot 45^\circ \\ &= \sin 90^\circ \\ &= 1\end{aligned}$$

$$\begin{aligned}2 \sin A &= 2 \sin 45^\circ \\ &= 2 \cdot \frac{1}{\sqrt{2}}\end{aligned}$$



(iii) $\sin 2A = 2 \sin A$ तब सत्य होता है,

जबकि A बराबर है :

~~(अ)~~ 0°

(ब) 30°

(स) 45°

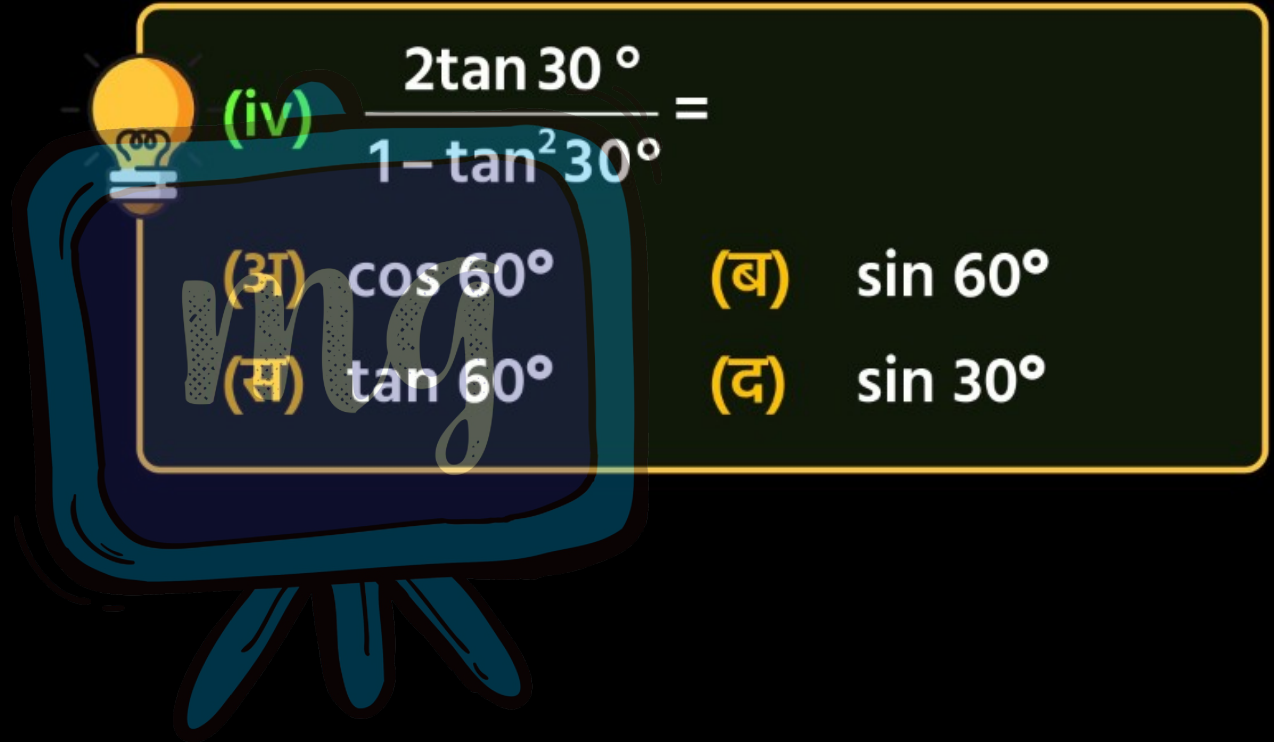
(द) 60°

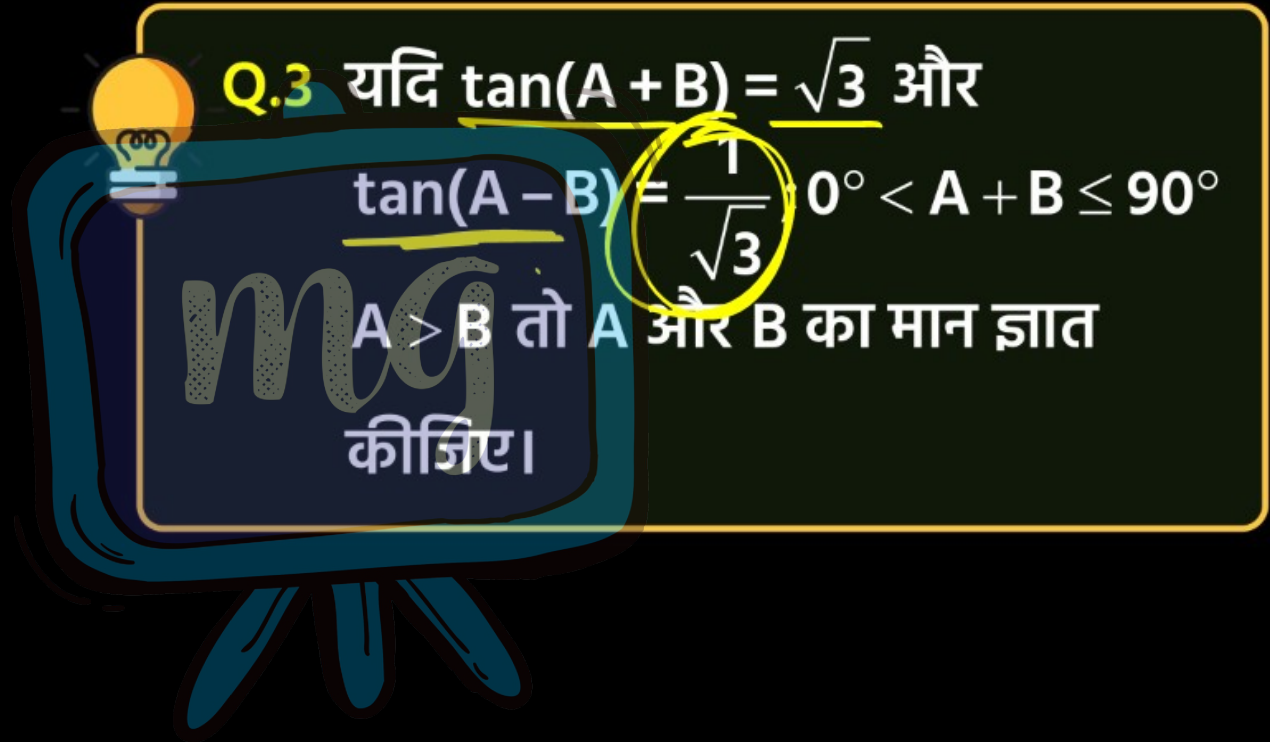
$$\sin 2A = 2 \sin A$$

$$A = 0^\circ$$

$$\begin{aligned}\sin 2A &= \sin 2 \cdot 0^\circ = \sin 0^\circ = 0 \\ 2 \sin A &= 2 \sin 0^\circ = 2 \cdot 0 = 0\end{aligned}$$

H.W.





Q.3 यदि $\tan(A + B) = \sqrt{3}$ और

$\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B \leq 90^\circ$

A > B तो A और B का मान ज्ञात कीजिए।

$$\tan(A+B) = \sqrt{3}$$

$$\tan(A+B) = \tan 60^\circ$$

$$A+B = 60^\circ \text{ --- (i)}$$

समी. (i) व (ii) को

जोड़ने पर

$$A+B = 60^\circ$$

$$A-B = 30^\circ$$

$$2A = 90^\circ$$

$$A = 45^\circ \checkmark$$

$$\tan(A-B) = \frac{1}{\sqrt{3}}$$

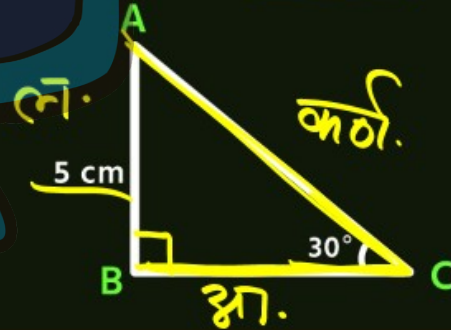
$$\tan(A-B) = \tan 30^\circ$$

$$A-B = 30^\circ \text{ --- (ii)}$$



उदाहरण-6

$\triangle ABC$ में जिसका कोण B समकोण है,
 $AB = 5 \text{ cm}$ और $\angle ACB = 30^\circ$ (देखिए
आकृति)। भुजाओं BC और AC की लंबाइयाँ
ज्ञात करें।



$\triangle ABC$

$\angle B = 90^\circ$, $AB = 5 \text{ cm}$

$$\cos C = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 30^\circ = \frac{BC}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{BC}{105}$$

$$\underline{BC = 5\sqrt{3} \text{ cm}}$$

$$\sin C = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{5}{AC}$$

$$\underline{AC = 10 \text{ cm}}$$

उदाहरण-7

$$\angle Q = 90^\circ$$

$$PQ = 3 \text{ cm}$$

$$PR = 6 \text{ cm}$$

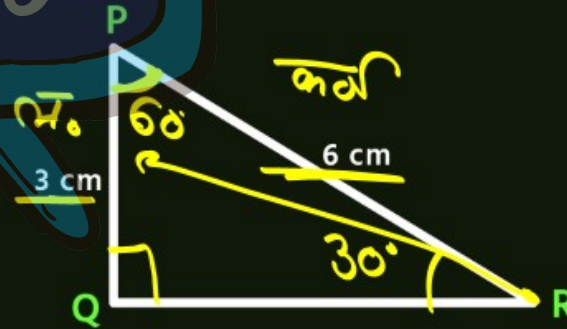
$$\sin R = \frac{\text{ल. को.}}{\text{कर्ण}}$$

$$\sin R = \frac{PQ}{PR}$$

$$\sin R = \frac{3}{6}$$

$$\sin R = \frac{1}{2}$$

$\triangle PQR$ में, जिसका कोण Q समकोण है (देखिए आकृति), $PQ = 3 \text{ cm}$ और $PR = 6 \text{ cm}$ है। $\angle QPR$ और $\angle PRQ$ ज्ञात कीजिए।





H.W.

उदाहरण-8



$$\text{यदि } \sin(A - B) = \frac{1}{2}, \cos(A + B) = \frac{1}{2}$$

$0^\circ < A + B \leq 90^\circ, A > B$, तो A और B ज्ञात कीजिए।