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कक्षा-12

सारणिक

(Determinants)

अध्याय - 4

भाग - 5

प्रश्नावली 4.3

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$$A_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix}$$

$$= -12 - 0 = -12$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = -(8 - 30)$$

$$= 22$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18$$

$$= 18$$

उदाहरण -11 : सारणिक $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ के अवयवों के उपसारणिक और सहखंड ज्ञात कीजिए और सत्यापित कीजिए कि $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$ है।

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LHS:

अब, $a_{11} A_{31} + a_{12} a_{32} + a_{13} \cdot a_{33}$

$= 2(-12) + (-3)(22) + 5(18)$

$= -24 - 66 + 90$

$= -90 + 90 = 0$ RHS

इति सिद्धम्।

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निम्नलिखित सारणिकों के अवयवों के उपसारिणक एवं सहखंड लिखिए।

1. (i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

$M_{11} = 3 \Rightarrow A_{11} = (-1)^{1+1} \cdot (3) = 3$

$M_{12} = 0 \Rightarrow A_{12} = (-1)^{1+2} \cdot (0) = 0$

$M_{21} = -4 \Rightarrow A_{21} = (-1)^{2+1} \cdot (-4) = 4$

$M_{22} = 2 \Rightarrow A_{22} = (-1)^{2+2} \cdot (2) = 2$

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$M_{11} = d \Rightarrow A_{11} = (-1)^{1+1} \cdot (d) = d$

$M_{12} = b \Rightarrow A_{12} = (-1)^{1+2} \cdot (b) = -b$

$M_{21} = c \Rightarrow A_{21} = (-1)^{2+1} \cdot (c) = -c$

$M_{22} = a \Rightarrow A_{22} = (-1)^{2+2} \cdot (a) = a$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

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$$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow A_{11} = (-1)^{1+1} \cdot (1)$$
$$= 1$$

$$M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \Rightarrow A_{12} = (-1)^{1+2} \cdot (0)$$
$$= 0$$

$$M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \Rightarrow A_{13} = (-1)^{1+3} \cdot (0) = 0$$

2. (i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \Rightarrow A_{21} = (-1)^{2+1} \cdot (0) = 0$$

$$M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow A_{22} = (-1)^{2+2} \cdot (1) = 1$$

$$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \Rightarrow A_{23} = (-1)^{2+3} \cdot (0) = 0$$

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$$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$$

$$A_{11} = (-1)^{1+1} \cdot (11) = 11$$

$$M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 \Rightarrow A_{12} = (-1)^{1+2} \cdot (6) = -6$$

$$M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 \Rightarrow A_{13} = (-1)^{1+3} \cdot (3) = 3$$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

$$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -4 \Rightarrow A_{21} = (-1)^{2+1} \cdot (-4) = 4$$

$$M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 \Rightarrow A_{22} = (-1)^{2+2} \cdot (2) = 2$$

$$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow A_{23} = (-1)^{2+3} \cdot (1) = -1$$

$$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = -20 \Rightarrow A_{31} = (-1)^{3+1} \cdot (-20) = -20$$

$$M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13 \Rightarrow A_{32} = (-1)^{3+2} \cdot (-13) = 13$$

$$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 \Rightarrow A_{33} = (-1)^{3+3} \cdot (5) = 5$$

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$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = (-)(9-16)$

$= 7$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (-)(10-3)$

$= -7$

अब, $\Delta = a_{21} \cdot A_{21} + a_{22} \cdot A_{22} + a_{23} \cdot A_{23}$

3. दूसरी पंक्ति के अवयवों के सहखंडों का प्रयोग करके $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ का मान ज्ञात कीजिए।

अत: $\Delta = (2)(7) + (0)(7) + (1)(-7)$

$= 14 + 0 - 7$

$\boxed{\Delta = 7}$

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$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-)(z-x) = x - z$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

4. तीसरे स्तंभ के अवयवों के सहखंडों का प्रयोग करके $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$. का मान ज्ञात कीजिए।

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अब, $\Delta = a_{13} \cdot A_{13} + a_{23} \cdot A_{23} + a_{33} \cdot A_{33}$

$\Rightarrow \Delta = yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + x^2z - z^2x + xy^2 - x^2y$

$= x^2z - x^2y + y^2x - y^2z + yz^2 - xz^2$

$= x^2(z-y) + y^2(x-z) + z^2(y-x)$.