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ई-पाठशाला कार्यक्रम

CLASS 10th MATHS

त्रिकोणमिति का परिचय

INTRODUCTION TO TRIGONOMETRY

RAPID REVISION



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Aim-100

$\sin\theta = \frac{1}{\text{cosec}\theta}$ | $\text{cosec}\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ | $\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ | $\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ | $\cot\theta = \frac{\cos\theta}{\sin\theta}$

कर्ण^2 = लं$_0^2$ + आ$_0^2$

कर्ण — 90° — कोण

Sin | cos | tan | cot | sec | cosec

LAL

KKA

cosec sec cot

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Q.1 यदि $\sin A = \frac{3}{5}$ हो, तब $\tan A + \cos A$ का मान ज्ञात कीजिए।

हल:-

$\sin A = \frac{3}{5} = \frac{\text{लं०}}{\text{कर्ण}}$

लं० = 3k, कर्ण = 5k, आ० = ?

पाइथागोरस सूत्र,

कर्ण² = लं०² + आ०²

$(5k)^2 = (3k)^2 + \text{आ०}^2$

$25k^2 = 9k^2 + \text{आ०}^2$

$25k^2 - 9k^2 = \text{आ०}^2$

$16k^2 = \text{आ०}^2$

$\sqrt{16k^2} = \text{आ०}$

$4k = \text{आ०}$

$$\tan A + \cos A = \frac{3}{4} + \frac{4}{5}$$

$$= \frac{15+16}{20}$$

$$= \frac{31}{20}$$



$$\tan A = \frac{\text{लं०}}{\text{आ०}} = \frac{3k}{4k}$$

$$\tan A = \frac{3}{4}$$

$$\cos A = \frac{\text{आ०}}{\text{कर्ण}} = \frac{4k}{5k}$$

$$\cos A = \frac{4}{5}$$

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M-I

Q.2 यदि tan A = 1 हो, तो 2 sin A cos A का मान ज्ञात कीजिए ।

हल:-

$\tan A = \frac{1}{1} = \frac{\text{ल.}}{\text{आ.}}$

ल. = k, आ. = k

$\text{कर्ण}^2 = \text{ल.}^2 + \text{आ.}^2$

$\text{कर्ण}^2 = k^2 + k^2$

$\text{कर्ण}^2 = 2k^2$

$\text{कर्ण} = \sqrt{2k^2} = \sqrt{2}k$

$\sin A = \frac{\text{ल.}}{\text{कर्ण}} = \frac{k}{\sqrt{2}k}$

$\sin A = \frac{1}{\sqrt{2}}$

$\cos A = \frac{\text{आ.}}{\text{कर्ण}} = \frac{k}{\sqrt{2}k}$

$\cos A = \frac{1}{\sqrt{2}}$

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M - II

$\tan A = 1$

$\tan A = \tan 45^\circ$

$\boxed{A = 45^\circ}$

$2 \sin A \cos A$

$= 2 \sin 45^\circ \cos 45^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$


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Q.3 यदि $\angle A$ और $\angle B$ न्यून कोण हो, जहाँ $\cos A = \cos B$, तो दिखाइए कि $\angle A = \angle B$

हल :-

$\cos B = \frac{BC}{AB}$

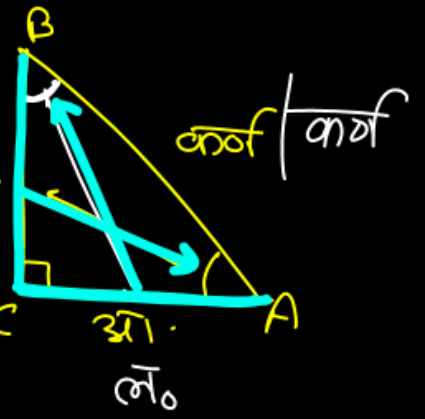
$\because \cos A = \cos B$

$\frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

समद्विबाहु Δ में समान भुजाओं के सम्मुख कोण समान होते हैं।

$\angle A = \angle B$



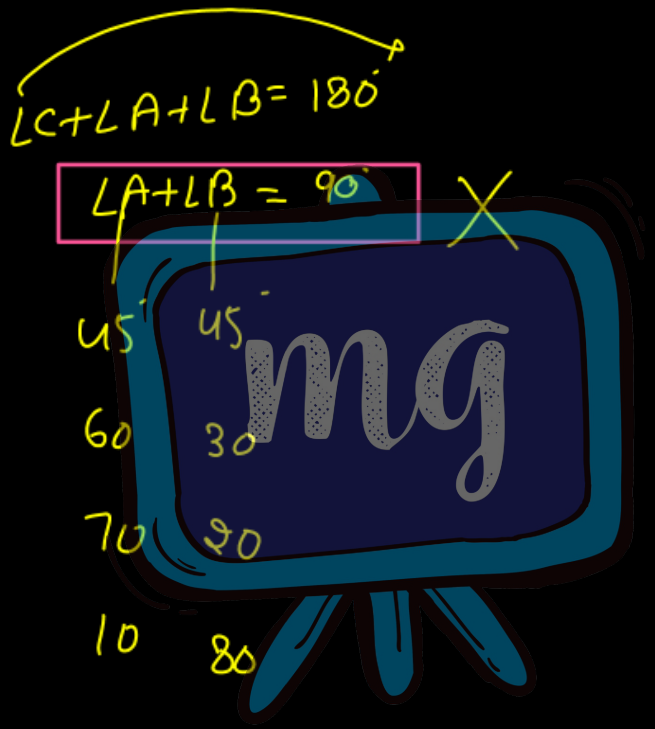
आ./ल. | कर्ण/कर्ण

$\cos A = \frac{\text{आ.}}{\text{कर्ण}}$

$\cos A = \frac{AC}{AB}$

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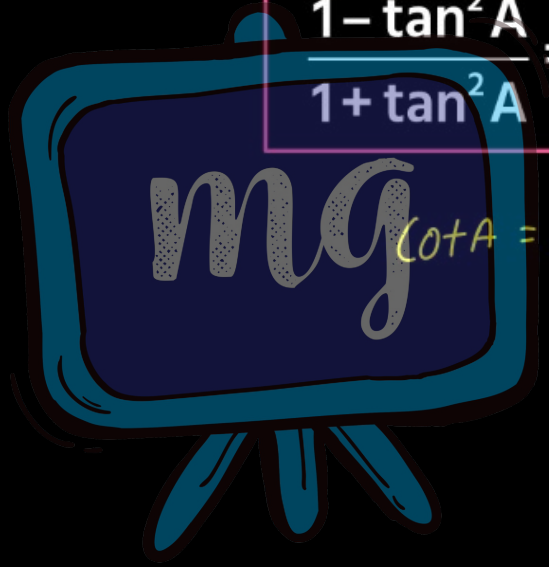
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H.W.

Q.4 यदि $3 \cot A = 4$, तो जाँच कीजिए कि

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$ है या नहीं।



$\cot A = \frac{4}{3} = \frac{\text{आ.}}{\text{ल.}}$

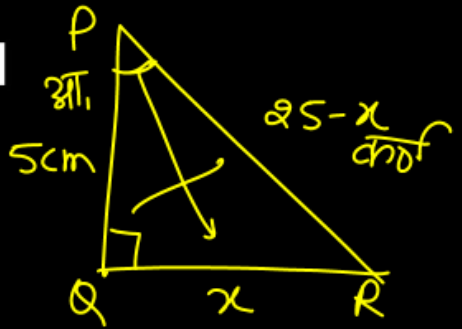

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$\{(a-b)^2 = a^2+b^2-2ab\}$

Q.5 Δ PQR में, जिसका कोण Q समकोण है, PR + QR = 25 cm और PQ = 5 cm है। sin P, cos P और tan P के मान ज्ञात कीजिए।



हल:- PQ = 5 cm

PR + QR = 25 cm

QR = x cm

PR + x = 25

PR = (25-x) cm

कर्ण² = स.² + आ.²

$PR^2 = QR^2 + PQ^2$

$(25-x)^2 = x^2 + 5^2$

$(25)^2 + (x)^2 - 2\times25\times x = x^2 + 25$

$625 + x^2 - 50x = x^2 + 25$

$625 - 25 = 50x$

$600 = 50x$

$x = 12$ cm

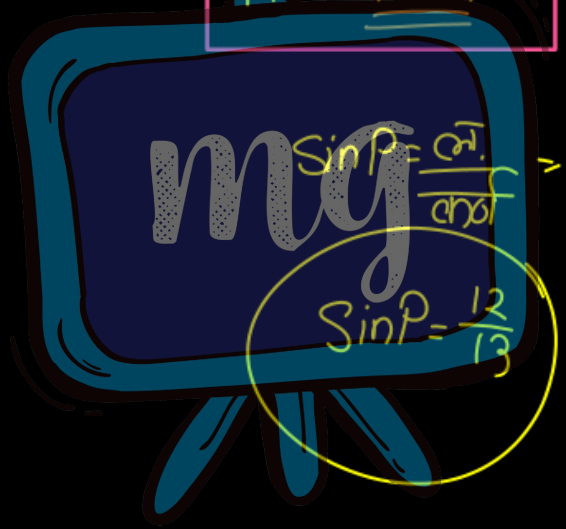
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$\frac{4AL}{K/KA}$

$PR = 25 - 12$

$PR = 13cm$

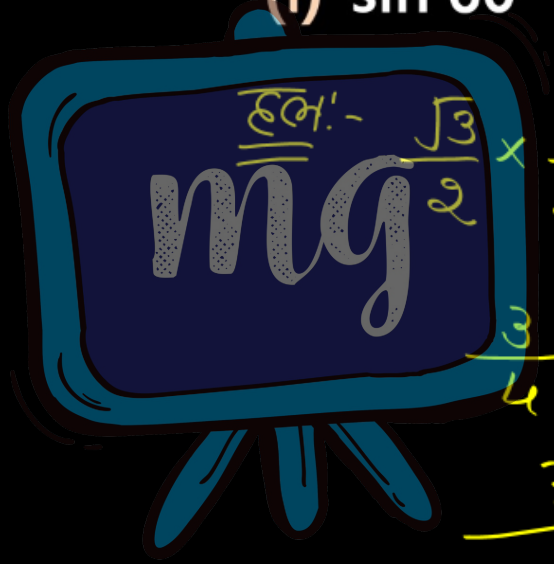


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Q.6 निम्नलिखित के मान ज्ञात कीजिए

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$



हल:-

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$\frac{3}{4} + \frac{1}{4}$

$\frac{3+1}{4} = \frac{4}{4} = 1$

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(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Sol:- $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$2 \times 1 = 2$



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(iii) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

हल :- $\dfrac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} \times \frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$

$= \dfrac{\frac{1}{2} + 1 - \frac{2\sqrt{3}}{3}}{\frac{2\sqrt{3}}{3} + \frac{1}{2} + 1}$

$= \dfrac{\frac{3+6-4\sqrt{3}}{6}}{\frac{4\sqrt{3}+3+6}{6}}$

$= \dfrac{9-4\sqrt{3}}{9+4\sqrt{3}} \times \dfrac{(9-4\sqrt{3})}{9-4\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sec 30° = \frac{2}{\sqrt{3}}$

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$$= \frac{(9-4\sqrt{3})^2}{9^2-(4\sqrt{3})^2}$$

$$= \frac{9^2+(4\sqrt{3})^2-2\times 9\times 4\sqrt{3}}{81-16\times 3}$$

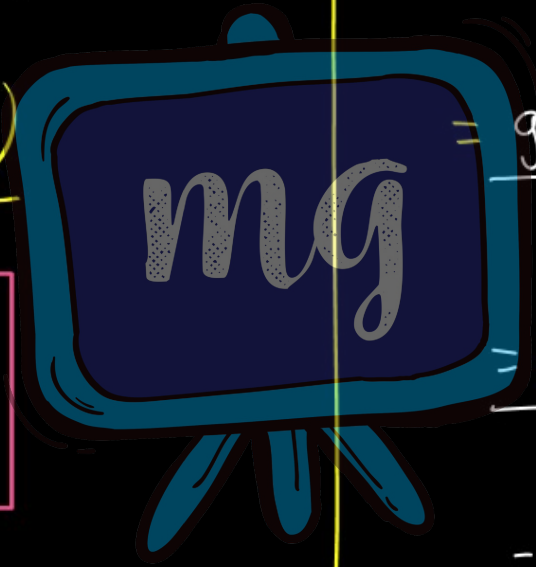
$$= \frac{81+16\times 3-72\sqrt{3}}{81-48}$$

$$= \frac{81+48-72\sqrt{3}}{33}$$

$$= \frac{129-72\sqrt{3}}{33}$$

$$= \frac{3(43-24\sqrt{3})}{33\ 11}$$

$$= \frac{43-24\sqrt{3}}{11}$$




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Q.7 सही विकल्प चुनिए और अपने विकल्प का औचित्य दीजिए :

(i) $\frac{2\tan 30°}{1+\tan^2 30°} =$

(अ) $\sin 60°$ (ब) $\cos 60°$

(स) $\tan 60°$ (द) $\sin 30°$

$$\frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$\frac{2\times 3\sqrt{3}}{2\times 4\times\sqrt{3}} = \frac{\sqrt{3}}{2}$$

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$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\tan 30^\circ}{\sec^2 30^\circ}$$



$$= \frac{2 \times \frac{\sin 30^\circ}{\cos 30^\circ}}{\frac{1}{\cos^2 30^\circ}}$$

$$= 2 \times \sin 30^\circ \times \cos 30^\circ$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \sin 60^\circ$$

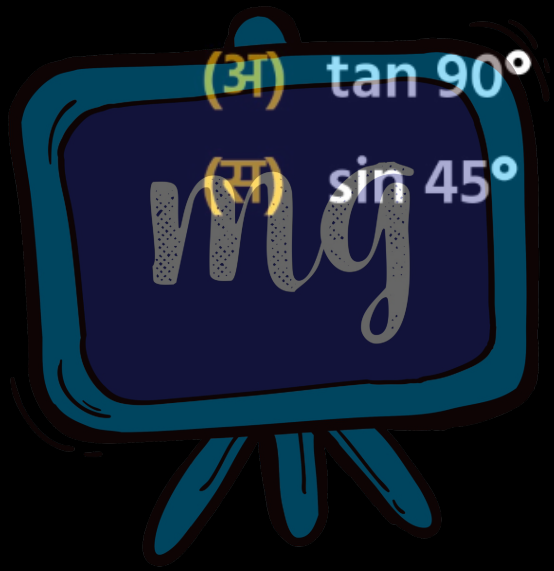
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(ii) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} = \dfrac{1-(1)^2}{1+(1)^2} = \dfrac{0}{1} = 0$

(अ) $\tan 90°$ (ब) 1

(स) $\sin 45°$ (द) 0



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$\angle Q = 90$

Q.8 $\triangle PQR$ में, जिसका कोण Q समकोण है (देखिए आकृति), PQ = 3 cm और PR = 6 cm है। $\angle QPR$ और $\angle PRQ$ ज्ञात कीजिए।

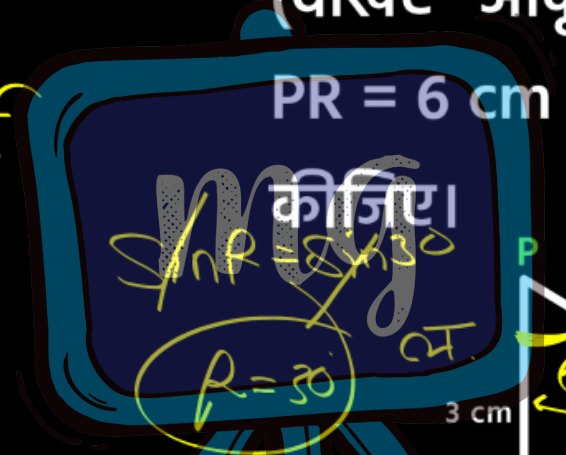
$\sin R = \frac{\text{सा.}}{\text{कर्ण}}$

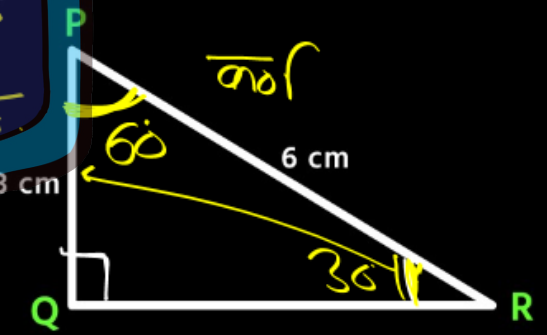
$\sin R = \frac{PQ}{PR}$

$\sin R = \frac{3}{6}$

$\sin R = \frac{1}{2} = \sin(30)$

$R = 30$





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Q.9 यदि $\tan(A+B)=\sqrt{3}$ और $\tan(A-B)=\frac{1}{\sqrt{3}}; 0^\circ < A+B \le 90^\circ$

$A > B$ तो A और B का मान ज्ञात कीजिए।

$\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60^\circ$

$A+B=60^\circ$ — (i)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\tan(A-B)=\tan 30^\circ$

$A-B=30^\circ$ — (ii)

A | B

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Q.10 सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए :

(i) $9 \sec^2 A - 9 \tan^2 A$ बराबर है :

(अ) 1 (ब) 9

(स) 8 (द) 10

(ii) (sec A+ tan A) (1 – sin A)

बराबर है :

(अ) sec A (ब) sin A

(स) cosec A (द) cos A



(iii) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ बराबर है :

(अ) $\sec^2 A$ (ब) -1

(स) $\cot^2 A$ (द) $\tan^2 A$

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Q.11 निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$



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(iii) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$



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(iv) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$



(v) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$



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(vi) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$



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$$\text{(vii)}\ (\text{cosec}A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

