

कक्षा - 10

गणित

अध्याय - 8

त्रिकोणमिति का परिचय

भाग - 3

केशव शर्मा

Aim-100

Sin/cos/tan/cot/sec/cosec



$$\sin = \frac{1}{\operatorname{cosec}}$$

$$\cos = \frac{1}{\sec}$$

$$\tan = \frac{1}{\cot}$$

$$\tan = \frac{\sin}{\cos}$$

$$\cot = \frac{\cos}{\sin}$$

Q. 7

यदि $\cot \theta = \frac{7}{8}$, तो

(i)
$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

(ii)
$$\frac{\cot 2\theta}{\cot^2 \theta}$$

का मान निकालिए?

हल:- $\cot \theta = \frac{7}{8} \left(\frac{A}{L} \right)$

(ii) $\cot^2 \theta = \left(\frac{7}{8} \right)^2 = \frac{49}{64}$

कर्ण $= \sqrt{113k^2}$
 $= \sqrt{113} k$

(i) $A = 7k, L = 8k, \text{ कर्ण} = ?$

$\text{कर्ण}^2 = \text{लंब}^2 + \text{आध}^2$
 $= (8k)^2 + (7k)^2$
 $= 64k^2 + 49k^2$

$\text{कर्ण}^2 = 113k^2$

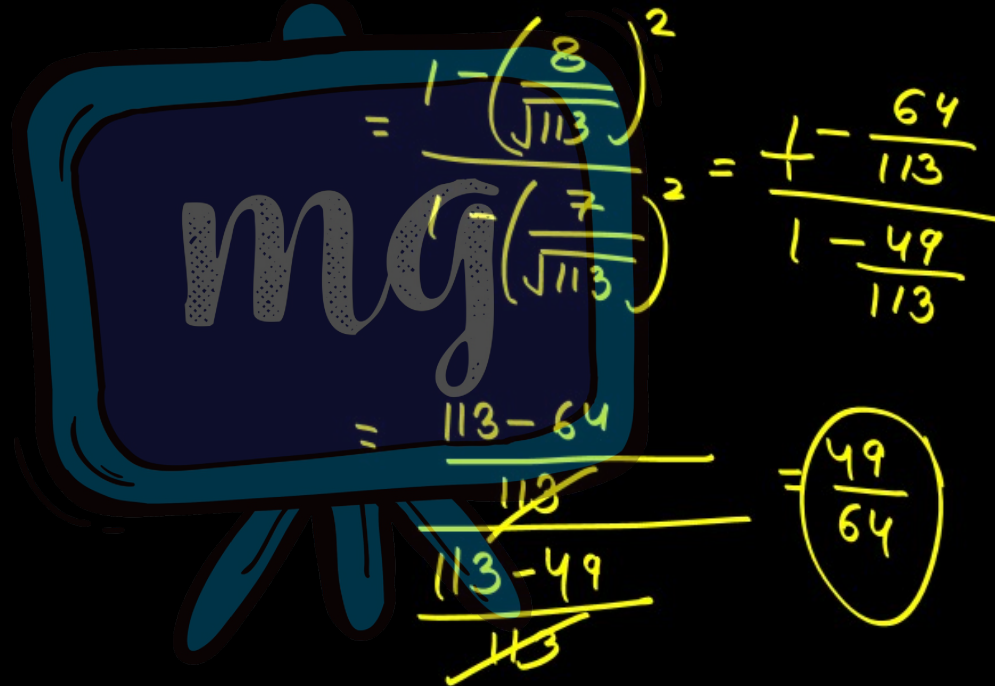
$$\sin \theta = \frac{L}{K} = \frac{8K}{\sqrt{113}K} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{A}{K} = \frac{7K}{\sqrt{113}K} = \frac{7}{\sqrt{113}}$$

$$(a+b)(a-b)$$

$$(i) \quad \frac{a+b}{1+\sin \theta} \cdot \frac{a-b}{1-\sin \theta} = \frac{a^2-b^2}{(1+\cos \theta)(1-\cos \theta)}$$

$$= \frac{1^2 - \sin^2 \theta}{1^2 - \cos^2 \theta} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$



The image shows a hand-drawn solution on a chalkboard. A large, stylized 'mg' watermark is centered on the board. The math is written in yellow chalk. The steps are as follows:

$$\begin{aligned} &= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} \\ &= \frac{113 - 64}{113 - 49} = \frac{49}{64} \end{aligned}$$

The final result, $\frac{49}{64}$, is circled in yellow.

Q. 8

$$\sin A = \frac{L}{K} = \frac{4K}{5K}$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{3K}{5K} = \frac{4K}{5K}$$

$$\cos A = \frac{4}{5}$$

यदि $3 \cot A = 4$, तो जाँच कीजिए कि

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A \text{ है या नहीं।}$$

हल $3 \cot A = 4$

$$\cot A = \frac{4}{3}$$

$$\tan A = \frac{1}{\cot A} = \frac{1}{\frac{4}{3}}$$

$$\tan A = \frac{3}{4} \left(\frac{L}{K} \right)$$

$$L = 3K, A = 4K, \\ \text{कर्ण} = 5K$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

LHS

$$\frac{1 - \tan^2 A}{1 + \tan^2 A}$$
$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$
$$= \frac{7}{25}$$

$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

LHS = RHS

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

RHS. $\cos^2 A - \sin^2 A$
 $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$$\frac{16}{25} - \frac{9}{25}$$

$$\frac{16 - 9}{25} = \left(\frac{7}{25}\right)$$

Q1. सिद्ध करें-

(i) $2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

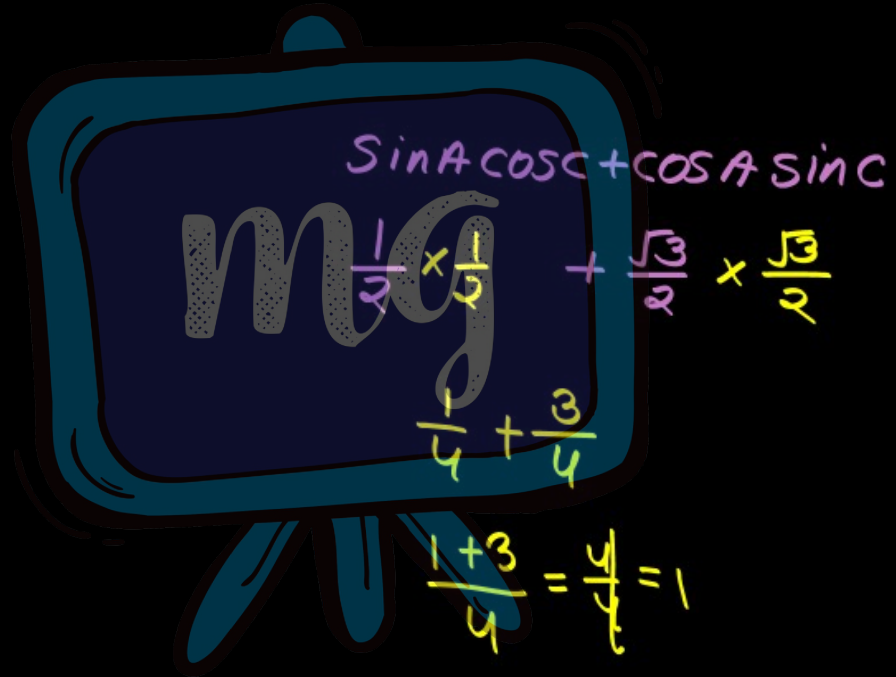
(ii) $1 - 2 \sin^2 A = 2 \cos^2 A - 1$

(iii) $(\tan A + \cot A)^2 = \left(\frac{\sin A + \cos A}{\cos A \sin A} \right)^2$

Q. 9

त्रिभुज ABC में, जिसका कोण B
समकोण है, यदि $\tan A = \frac{1}{\sqrt{3}}$, तो
निम्नलिखित के मान ज्ञात कीजिए :

(i) $\sin A \cos C + \cos A \sin C$


$$\sin A \cos C + \cos A \sin C$$
$$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$
$$\frac{1}{4} + \frac{3}{4}$$
$$\frac{1+3}{4} = \frac{4}{4} = 1$$

$$\tan A = \frac{1}{\sqrt{3}} = \frac{C}{A}$$

$$C = 1k$$

$$A = \sqrt{3}k$$

$$\text{Ans} = ?$$

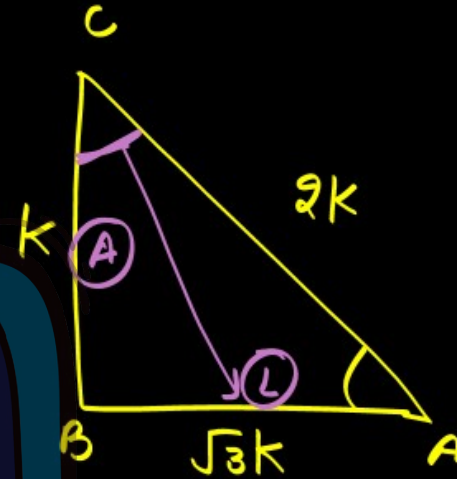
$$(\text{Ans})^2 = (1k)^2 + (\sqrt{3}k)^2$$

$$= k^2 + (\sqrt{3}k)^2$$

$$= k^2 + 3k^2$$

$$(\text{Ans})^2 = 4k^2$$

$$\begin{aligned} \text{Ans} &= \sqrt{4k^2} \\ &= \underline{\underline{2k}} \end{aligned}$$



$$\sin C = \frac{c}{k} = \frac{AB}{AC}$$

$$\sin C = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{1}{2}$$

$$\sin A = \frac{a}{k} = \frac{BC}{AC}$$

$$\sin A = \frac{K}{2K} = \frac{1}{2}$$

$$\cos A = \frac{b}{k} = \frac{AB}{AC}$$

$$\cos A = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

Q. 9

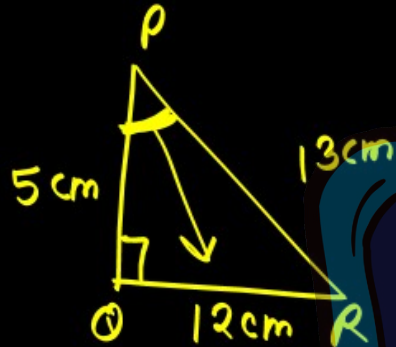
त्रिभुज ABC में, जिसका कोण B
समकोण है, यदि $\tan A = \frac{1}{\sqrt{3}}$, तो
निम्नलिखित के मान ज्ञात कीजिए :

(ii) $\cos A \cos C - \sin A \sin C$

$$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q. 10



$\triangle PQR$ में, जिसका कोण Q समकोण है, $PR + QR = 25$ cm और $PQ = 5$ cm है। $\sin P$, $\cos P$ और $\tan P$ के मान ज्ञात कीजिए।

$$PR = (25 - QR) \leftarrow PR + QR = 25 \text{ cm}, PQ = 5 \text{ cm}$$

$$PR^2 = PQ^2 + QR^2 \quad (\text{पाइथागोरस प्रमेय})$$

$$PR^2 = PQ^2 + QR^2$$

$$\begin{aligned}(25 - QR)^2 &= (5)^2 + QR^2 \\(25)^2 + (QR)^2 - 2 \times 25 \times QR &= 25 + QR^2 \\625 + QR^2 - 50QR &= 25 + QR^2 \\625 - 25 - 50QR &= 0 \\600 &= 50QR\end{aligned}$$

$$QR = \frac{600}{50}^{12}$$

$$\underline{QR = 12 \text{ cm}}$$

$$PR = 25 - 12 = \underline{\underline{13 \text{ cm}}}$$

$$\underline{\sin P = \frac{L}{K} = \frac{QR}{PR} = \frac{12}{13}}$$

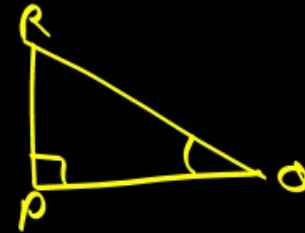
Ex. 2

यदि $\angle B$ और $\angle Q$ ऐसे न्यूनकोण हों
जिससे कि $\sin B = \sin Q$, तो सिद्ध
कीजिए कि $\angle B = \angle Q$

$$\sin B = \frac{AC}{BC}$$

$$\sin Q = \frac{PR}{RQ}$$

हल:-



ΔPQR में

$$QR^2 = PR^2 + PQ^2$$

$$QR^2 - PR^2 = PQ^2$$

$$\sqrt{QR^2 - PR^2} = PQ$$

$$\therefore \sin B = \sin Q$$

$$\therefore \frac{AC}{BC} = \frac{PR}{RQ}$$

$$\frac{AC}{PR} = \frac{BC}{RQ} = k \quad \text{--- (i)}$$

अब ΔABC में
पाइथागोरस प्रमेय से

$$BC^2 = AB^2 + AC^2$$

$$BC^2 - AC^2 = AB^2$$

$$AB = \sqrt{BC^2 - AC^2}$$

$$\frac{AB}{PQ} = \frac{\sqrt{BC^2 - AC^2}}{\sqrt{QR^2 - PR^2}}$$

$$\frac{AB}{PQ} = \frac{\sqrt{k^2 QR^2 - k^2 PR^2}}{\sqrt{QR^2 - PR^2}}$$

$$\left\{ \begin{array}{l} \frac{AC}{PR} = k \\ AC = kPR \end{array} \right\} \left| \begin{array}{l} \frac{BC}{RQ} = k \\ BC = kQR \end{array} \right.$$

$$\frac{AB}{PQ} = \frac{\sqrt{k^2 (QR^2 - PR^2)}}{\sqrt{QR^2 - PR^2}}$$

$$\frac{AB}{PQ} = \frac{\sqrt{(kQR)^2 - (kPR)^2}}{\sqrt{QR^2 - PR^2}}$$

SSS समरूपता के

$$\triangle ABC \sim \triangle PQR$$

$$\angle B = \angle Q$$

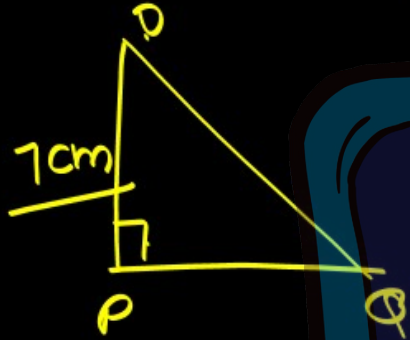
$$\frac{AB}{PQ} = \frac{\sqrt{QR^2 - PR^2}}{\sqrt{QR^2 - PR^2}}$$

$$\frac{AB}{PQ} = 1 \quad (ii)$$

अतः (i) व (ii) से

$$\frac{AC}{PR} = \frac{BC}{QR} = \frac{AB}{PQ}$$

Ex. 5



$$OQ^2 = OP^2 + PQ^2$$

$\triangle OPQ$ में, जिसका कोण P समकोण है, $OP = 7\text{cm}$ और $OQ - PQ = 1\text{cm}$
sin Q और cos Q के मान ज्ञात कीजिए

$$OQ = 1 + PQ$$

11.10.