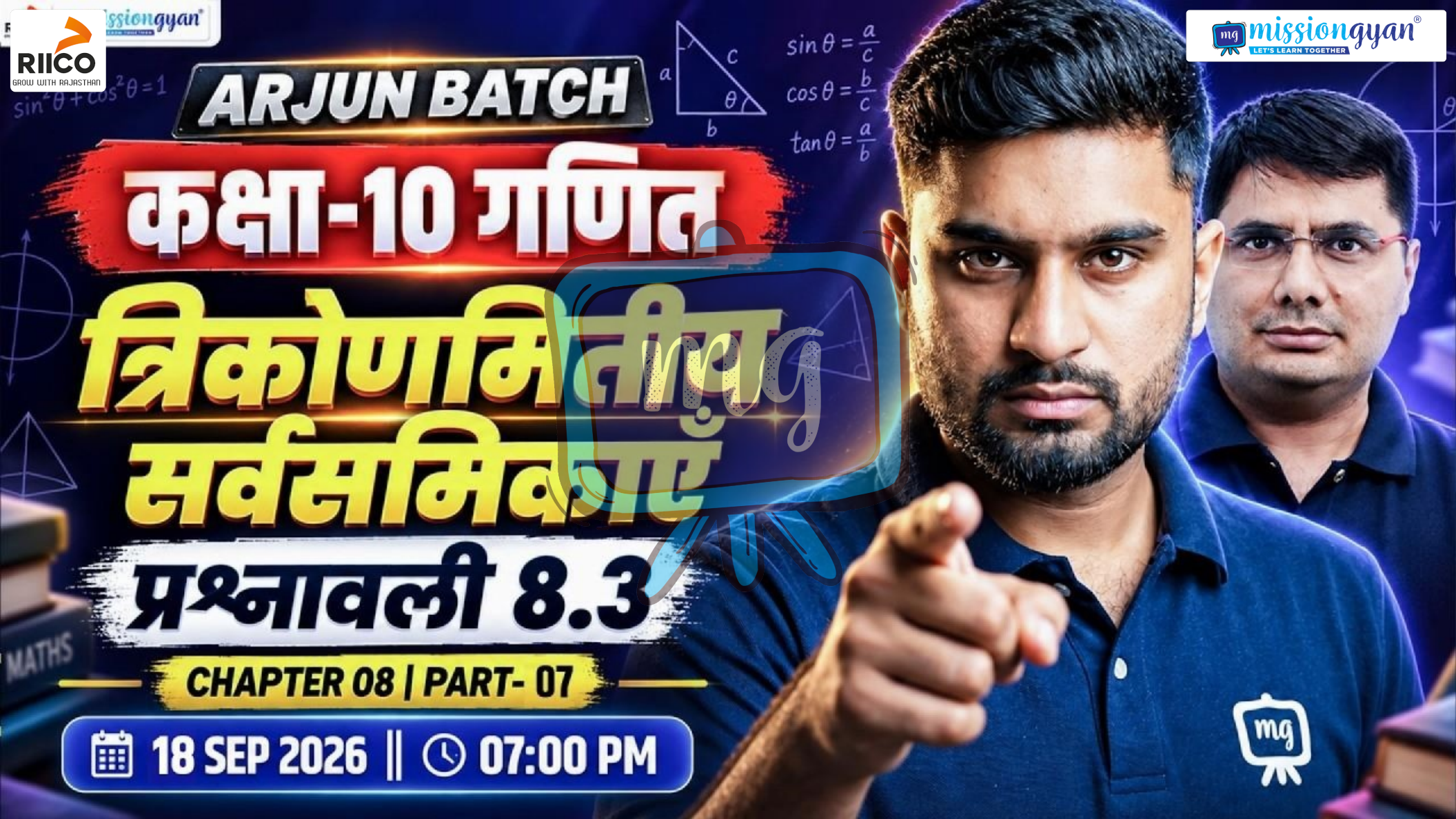

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ARJUN BATCH
कक्षा-10 गणित
त्रिकोणमिति
सर्वसमिकाएँ
प्रश्नावली 8.3
CHAPTER 08 | PART- 07
18 SEP 2026 || 07:00 PM
MATHS

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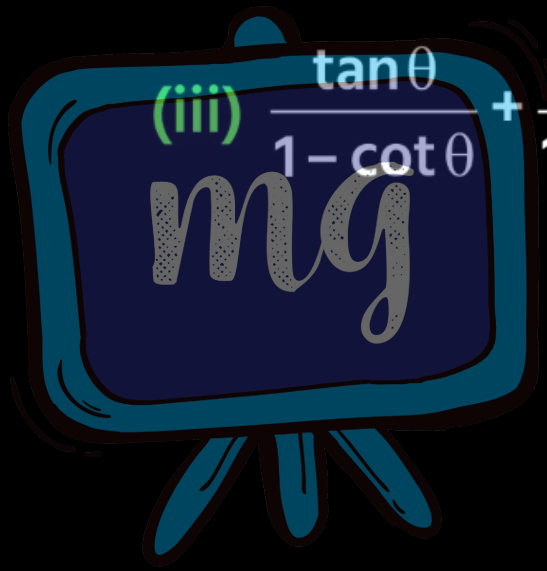
Q.4 निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है :

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

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(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

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(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

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$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

(1) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

(2) $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

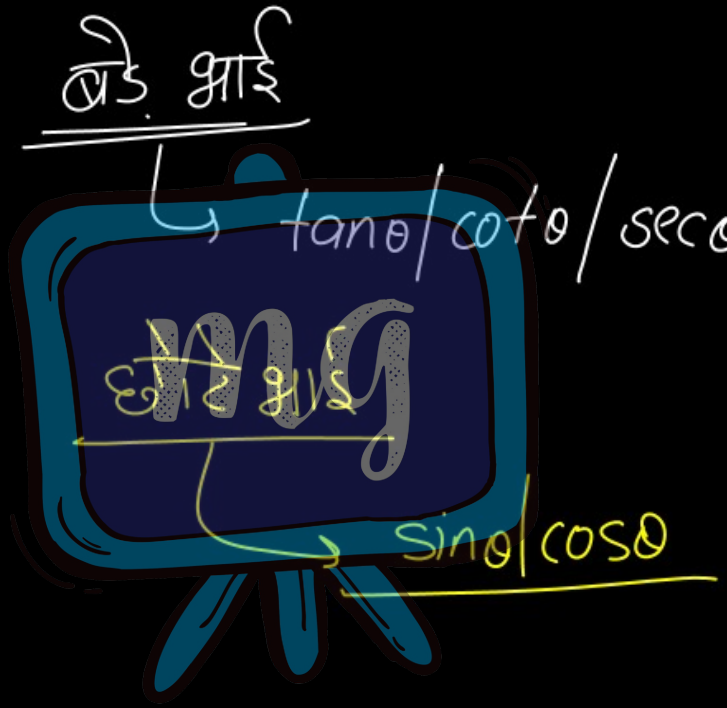
(3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

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(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

Sol. LHS

$= \dfrac{1+\sec A}{\sec A}$

$= \dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$\left\{\sec A = \dfrac{1}{\cos A}\right\}$

$= \dfrac{\left(\frac{\cos A + 1}{\cos A}\right)}{\frac{1}{\cos A}}$

$= \dfrac{(\cos A + 1)\cos A}{\cos A}$

$= 1 + \cos A$

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$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$1 = 1^2$

$a^2 - b^2 = (a+b)(a-b)$

RHS. $\dfrac{\sin^2 A}{1-\cos A}$

$= \dfrac{1^2 - \cos^2 A}{1 - \cos A}$

$= \dfrac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$

$= 1 + \cos A$

LHS = RHS

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$\frac{1}{\sec A + \tan A}$

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$\sec A + \tan A$

$\frac{1}{\cos} \quad \frac{\sin}{\cos}$

$\text{cosec}\, A = \frac{1}{\sin A}$

$\cot A = \frac{\cos A}{\sin A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}\, A + \cot A$

हल :- LHS.

$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

अंश और हर में $\sin A$ का भाग देने पर,

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$$= \frac{\cot A + \operatorname{cosec} A - 1}{1 + \cot A - \operatorname{cosec} A} \quad k^*$$

$$\{1 = \operatorname{cosec}^2 A - \cot^2 A\}$$

$$= \frac{\operatorname{cosec} A + \cot A - \{\operatorname{cosec}^2 A - \cot^2 A\}}{1 + \cot A - \operatorname{cosec} A}$$

$$= \frac{\cos A - \sin A + 1}{\sin A}$$

$$\frac{\cos A + \sin A - 1}{\sin A}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\text{cosec}A + \cot A - [(\text{cosec}A + \cot A)(\text{cosec}A - \cot A)]}{1 + \cot A - \text{cosec}A}$$

$$= \frac{(\text{cosec}A + \cot A)[1 - (\text{cosec}A - \cot A)]}{1 + \cot A - (\text{cosec}A)}$$

$$= \frac{(\text{cosec}A + \cot A)[1 - \text{cosec}A + \cot A]}{1 + \cot A - \text{cosec}A}$$

$$= \text{cosec}A + \cot A$$

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(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

हल- LHS

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{\sqrt{(1+\sin A)^2}}{\sqrt{1^2-\sin^2 A}}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A}}$$

हर का परिमेयकरण करने पर

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$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$= \sec A + \tan A$

LHS = RHS

$= \frac{1 + \sin A}{\sqrt{1 - \sin^2 A}}$

$= \frac{1 + \sin A}{\sqrt{\cos^2 A}}$

$= \frac{1 + \sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

प्रश्नावली 8.3

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Sol:- LHS :

$= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

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$$= \tan\theta \left(\frac{1 - 2\sin^2\theta}{2\cos^2\theta - 1} \right)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= \tan\theta \left[\frac{1 - 2(1 - \cos^2\theta)}{2\cos^2\theta - 1} \right]$$

$$= \tan\theta \left[\frac{2\cos^2\theta - 1}{2\cos^2\theta - 1} \right]$$

$$= \tan\theta \left[\frac{1 - 2 + 2\cos^2\theta}{2\cos^2\theta - 1} \right]$$

$$= \tan\theta$$

LHS = RHS

$$= \tan\theta \left[\frac{-1 + 2\cos^2\theta}{2\cos^2\theta - 1} \right]$$

$(I+II)^2 = I^2 + II^2 + 2II$

प्रश्नावली 8.3

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

हल :- LHS.

$= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \times \sin A \times \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \sec A \cos A$

$= \sin^2 A + \cos^2 A + 2 \sin A \frac{1}{\sin A} + 2 \sec A \times \frac{1}{\sec A} + \sec^2 A + \operatorname{cosec}^2 A$

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$= 1+2+2+\sec^2 A+\operatorname{cosec}^2 A$

$= 5+1+\tan^2 A+1+\cot^2 A$

$= 7+\tan^2 A+\cot^2 A$

LHS = RHS

प्रश्नावली 8.3

(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

LHS $= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \dfrac{\sin A}{1}\right)\left(\dfrac{1}{\cos A} - \dfrac{\cos A}{1}\right)$

$= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

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$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1}$$

$$= \sin A \cos A$$

LHS = RHS

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \sin A \cos A$$

RHS

$$= \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

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$$\frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \times$$



$$\frac{\sin A \cos A}{\sin^2 A \cos A}$$

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(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

हल :- $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\left(\frac{1}{\cot A}\right)^2}{1+\cot^2 A}$

$= \frac{1+\frac{1}{\cot^2 A}}{1+\cot^2 A}$

$= \frac{1+1^2}{\cot^2 A}$

$= \frac{\cot^2 A + 1}{\cot^2 A}$

$= \frac{\cot^2 A + 1}{1+\cot^2 A}$

$= \frac{1+\cot^2 A}{\cot^2 A(1+\cot^2 A)}$

$= \frac{1}{\cot^2 A} = \tan^2 A$

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$$= \frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{1+\tan^2 A}{1+\left(\frac{1}{\tan A}\right)^2}$$