ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 6

प्रश्नावली 5.1

By : Rajendra Yadav Sir

ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 5.1

$f(x) = \sin x + \cos x$

21. निम्नलिखित फलनों के सांतत्य पर विचार कीजिए:

(a) f(x) = sin x + cos x

LHL :

$$\lim_{h \to 0} f(a-h) = \lim_{h \to 0} [\sin(a-h) + \cos(a-h)]$$

$$= \lim_{h \to 0} [\sin a \cdot \cos h - \cos a \sin h + \cos a \cdot \cos h + \sin a \cdot \sin h]$$

$$= \sin a - 0 + \cos a + 0$$

$$= \sin a + \cos a$$

ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

RHL : $\lim_{h \to 0} f(a+h) = \lim_{h \to 0} [\sin(a+h) + \cos(a+h)]$

$= \lim_{h \to 0} [\sin a \cdot \cos h + \cos a \cdot \sin h + \cos a \cdot \cos h - \sin a \cdot \sin h]$

$= \sin a + 0 + \cos a - 0$

$= \sin a + \cos a$

तथा $f(a) = \sin a + \cos a$

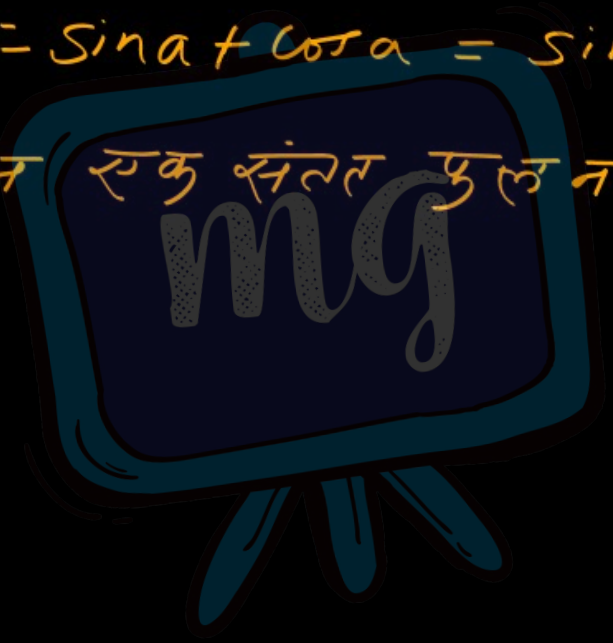
ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

अतः $LHL = RHL = f(a)$

$\sin a + \cos a = \sin a + \cos a = \sin a + \cos a$

$\Rightarrow$ यह फलन एक संतत फलन है।



ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

प्रश्नावली 5.1

(b) $f(x) = \sin x - \cos x$



ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

प्रश्नावली 5.1

(c) $f(x) = \sin x \cdot \cos x$

LHL:

$$\lim_{h \to 0} f(a-h) = \lim_{h \to 0} \sin(a-h) \cdot \cos(a-h)$$

$$= \lim_{h \to 0} [(\sin a \cdot \cos h - \cos a \cdot \sin h)(\cos a \cdot \cos h + \sin a \cdot \sin h)]$$

$$= \sin a \cdot \cos a$$

ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

RHL:

$$\lim_{h \to 0} f(a+h) = \lim_{h \to 0} \sin(a+h) \cdot \cos(a+h)$$

$$= \lim_{h \to 0} \left[(\sin a \cdot \cos h + \cos a \sin h)(\cos a \cdot \cos h - \sin a \cdot \sin h) \right]$$

$$= \sin a \cdot \cos a$$

$$f(a) = \sin a \cdot \cos a$$

$\Rightarrow$ LHL = RHL = $f(a)$ $\Rightarrow$ सतत

ipca
A dose of life

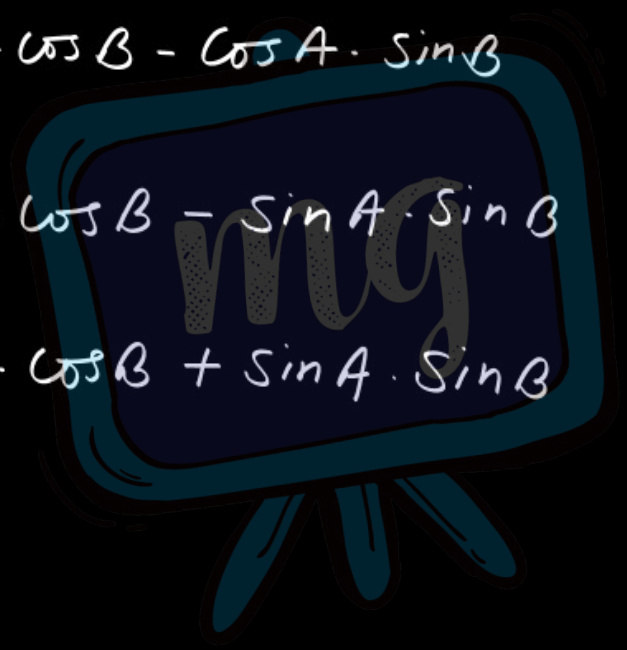
missiongyan®
LET'S LEARN TOGETHER

1) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

2) $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

3) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

4) $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$



ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 5.1

(ii)

$f(x) = \operatorname{cosec} x$

$f(x) = \frac{1}{\sin x}$

Cos x Cosec x Sec x

22. Cosine, cosecant, secant और cotangent फलनों के सांतत्य पर विचार कीजिए।

Cot x

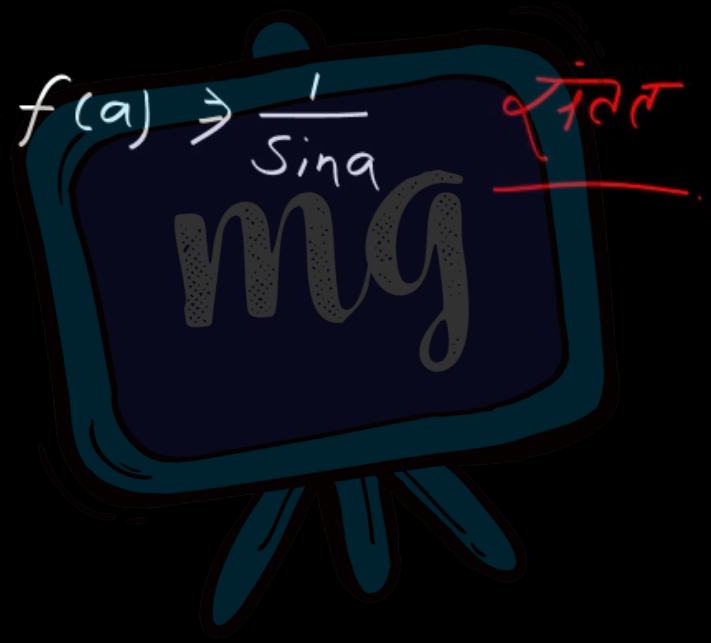
LHL: $\lim_{h \to 0} f(a-h) = \lim_{h \to 0} \frac{1}{\sin(a-h)} = \lim_{h \to 0} \frac{1}{\sin a \cdot \cos h - \cos a \cdot \sin h}$

$= \frac{1}{\sin a}$

RHL: $\lim_{h \to 0} f(a+h) = \lim_{h \to 0} \frac{1}{\sin(a+h)} = \lim_{h \to 0} \frac{1}{\sin a \cdot \cos h + \cos a \cdot \sin h} = \frac{1}{\sin a}$

$$f(a) = \frac{1}{\sin a}$$

$$\Rightarrow \quad LHL = RHL = f(a) \Rightarrow \frac{1}{\sin a}$$ परिमित



(iv) $f(x) = \cot x = \frac{\cos x}{\sin x}$

LHL: $\lim_{h \to 0} f(a-h) = \lim_{h \to 0} \frac{\cos(a-h)}{\sin(a-h)} = \lim_{h \to 0} \left[\frac{\cos a \cdot \cos h + \sin a \cdot \sin h}{\sin a \cdot \cos h - \cos a \cdot \sin h} \right]$

$= \frac{\cos a}{\sin a}$

RHL: $\lim_{h \to 0} f(a+h) = \lim_{h \to 0} \frac{\cos(a+h)}{\sin(a+h)} = \lim_{h \to 0} \left[\frac{\cos a \cdot \cos h - \sin a \cdot \sin h}{\sin a \cdot \cos h + \cos a \cdot \sin h} \right]$

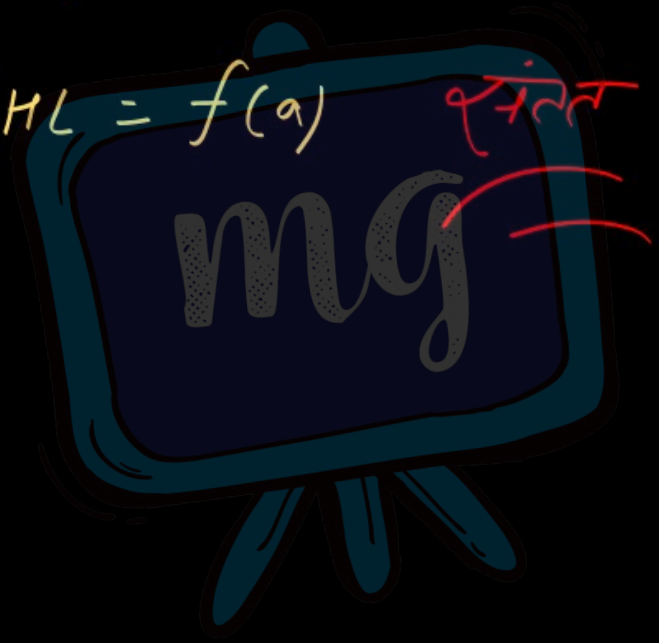
$= \frac{\cos a}{\sin a}$

ipca
A dose of life

missiongyan®
LET'S LEARN TOGETHER

तथा $f(a) = \frac{\cos a}{\sin a}$

$\Rightarrow$ LHL = RHL = $f(a)$ एवंतत



ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 5.1

23. f के सभी असांतत्यता के बिंदुओं को ज्ञात कीजिए, जहाँ

$$f(x)=\begin{cases}\dfrac{\sin x}{x}, & \text{यदि } x<0\\ x+1, & \text{यदि } x\geq 0\end{cases}$$

LHL:

$$\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\left(\frac{\sin(0-h)}{(0-h)}\right)$$

$$=\lim_{h\to 0}\left(\frac{-\sin h}{-h}\right)$$

$$=\lim_{h\to 0}\left(\frac{\sin h}{h}\right)=1$$

ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

RHL: $\lim_{h \to 0} f(0+h) = \lim_{h \to 0} [(0+h)+1] = 1$

$f(0) = 0+1 = 1$

$\Rightarrow$ LHL = RHL = $f(0)$ सतत
(at $x=0$) (at $x=0$)

यदि $x<0$ हो: $\frac{\sin k}{k}$ सतत

यदि $x>0$ हो: $k+1$ सतत

ipca
A dose of life

missiongyan
LET'S LEARN TOGETHER

प्रश्नावली 5.1

24. निर्धारित कीजिए कि फलन f

$$f(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{यदि } x\neq 0 \\ 0, & \text{यदि } x=0 \end{cases}$$

द्वारा परिभाषित एक संतत फलन है।

LHL = RHL = $f(a)$

संतत