कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 16

प्रश्नावली 5.7

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उदाहरण–37 : यदि $y = \sin^{-1} x$ है तो दर्शाइए कि

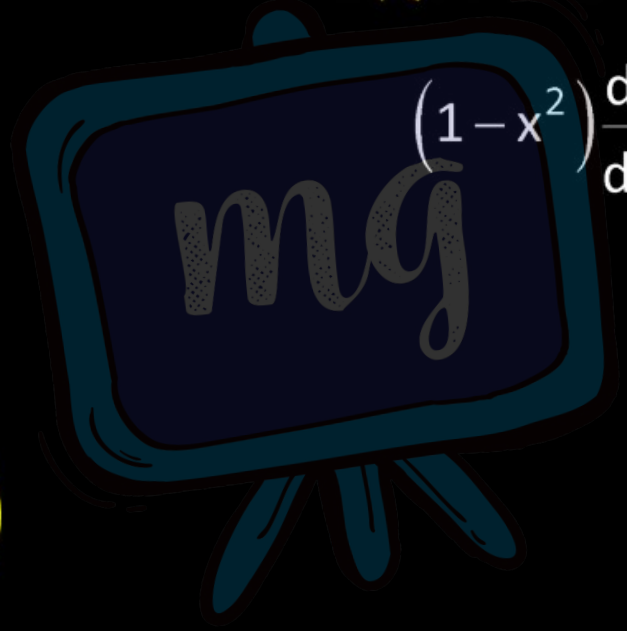
$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$ है।

$$y = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right)$$



$$\frac{d^2y}{dx^2} = -\frac{1}{2}\,\frac{1}{(1-x^2)}\,\frac{1}{\sqrt{1-x^2}}\,(-2x)$$

अत :

LHS : $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$

$$\Rightarrow (1-x^2)\left[-\frac{1}{2}\,\frac{1}{(1-x^2)}\,\frac{+2x}{\sqrt{1-x^2}}\right] - x\,\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = 0 \quad \text{RHS}$$

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प्रश्न संख्या 1 से 10 तक में दिए फलनों के द्वितीय कोटि के अवकलज ज्ञात कीजिए।

1. $x^2 + 3x + 2$

$$y = x^2 + 3x + 2$$

$$\Rightarrow \frac{dy}{dx} = 2x + 3$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(2x + 3)$$

$$= 2$$

$$\boxed{\frac{d^2y}{dx^2} = 2}$$

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2. x^{20}

$y = x^{20}$

$\Rightarrow \frac{dy}{dx} = 20x^{19}$

$\frac{d}{dx}x^n = nx^{n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}(20x^{19})$

$= 20 \times 19x^{18}$

$\Rightarrow \frac{d^2y}{dx^2} = 380x^{18}$

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3. x . cos x

$y = x \cos x$

$\Rightarrow \frac{dy}{dx} = x(-\sin x) + \cos x \cdot 1$

$\frac{d}{dx}(u \cdot v) = uv' + vu'$

$\frac{dy}{dx} = -x \sin x + \cos x$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}(-x \sin x + \cos x)$

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$$= \frac{d}{dx}(-x \sin x + \cos x)$$

$$= -x \cos x - \sin x \cdot 1 - \sin x$$

$$\frac{d^2y}{dx^2} = -x \cos x - 2 \sin x$$



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4. log x

$y = \log x$

$\frac{dy}{dx} = \frac{1}{x}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left(\frac{1}{x}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(x^{-1})$

$= -1 \cdot x^{-1-1}$

$= -x^{-2}$

$\boxed{\frac{d^2y}{dx^2} = -\frac{1}{x^2}}$

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5. $x^3 \log x$

$$y = x^3 \log x$$

$$\frac{dy}{dx} = x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2$$

$$\frac{dy}{dx} = x^2 + 3x^2 \log x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(x^2 + 3x^2 \log x)$$

$$\frac{d^2y}{dx^2} = 2x + 3\left(x^2 \cdot \frac{1}{x} + \log x \cdot 2x\right)$$

$$= 2x + 3x + 6x \log x$$

$$= 5x + 6x \log x$$

$$\boxed{\frac{d^2y}{dx^2} = x(5 + 6 \log x)}$$

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6. $e^x \sin 5x$

$$y = e^x \sin 5x$$

$$\frac{dy}{dx} = e^x \cdot \cos 5x\,(5) + \sin 5x \cdot e^x$$

$$\frac{dy}{dx} = 5e^x \cos 5x + e^x \sin 5x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(5e^x \cos 5x + e^x \sin 5x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(5e^x \cos 5x + e^x \sin 5x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 5e^x(-\sin 5x)(5) + 5\cos 5x \cdot e^x + e^x \cdot \cos 5x(5) + \sin 5x \cdot e^x$$

$$= -25e^x \sin 5x + 10e^x \cos 5x + e^x \sin 5x$$

$$\frac{d^2y}{dx^2} = 10e^x \cos 5x - 24e^x \sin 5x$$

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7. $e^{6x} \cos 3x$

$$y = e^{6x} \cdot \cos 3x$$

$$\frac{dy}{dx} = e^{6x}(-\sin 3x)(3) + \cos 3x \cdot e^{6x}(6)$$

$$\frac{dy}{dx} = -3e^{6x} \sin 3x + 6e^{6x} \cos 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -3\left[e^{6x} \cdot \cos 3x\,(3) + \sin 3x \cdot e^{6x}(6)\right] + 6\left[e^{6x}(-\sin 3x)(3) + \cos 3x \cdot e^{6x}(6)\right]$$

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$$\frac{d^2y}{dx^2} = -9e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x + 36e^{6x}\cos 3x$$

$$\boxed{\frac{d^2y}{dx^2} = 27e^{6x}\cos 3x - 36e^{6x}\sin 3x}$$

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8. $\tan^{-1} x$

$y = \tan^{-1} x$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(1+x^2)^{-1}$

$= -1(1+x^2)^{-1-1}(2x)$

$= \frac{-1}{(1+x^2)^2}(2x)$

$\frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2}$

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9. log (log x)

$y = \log(\log x)$

$\frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x \log x}$

$\frac{d^2y}{dx^2} = \frac{(x \log x)(0) - 1\left(x \cdot \frac{1}{x} + \log x \cdot 1\right)}{(x \log x)^2}$

$\boxed{\frac{d^2y}{dx^2} = \frac{-(1 + \log x)}{(x \log x)^2}}$