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कक्षा-12

सांतत्य तथा अवकलनीयता

(Continuity and Differentiability)

अध्याय - 5

भाग - 13

प्रश्नावली 5.5

By : Rajendra Yadav Sir

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प्रश्नावली 5.5

$y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ——— ①

अब, $u = (\sin x)^x$

$\log u = \log (\sin x)^x$

8. $(\sin x)^x + \sin^{-1}\sqrt{x}$

$\Rightarrow \log u = x \log(\sin x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x \times \frac{1}{\sin x} \times \cos x + \log \sin x \cdot 1$

$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x]$ ——— ②

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इसीप्रकार,

$$v = \sin^{-1}\sqrt{x}$$

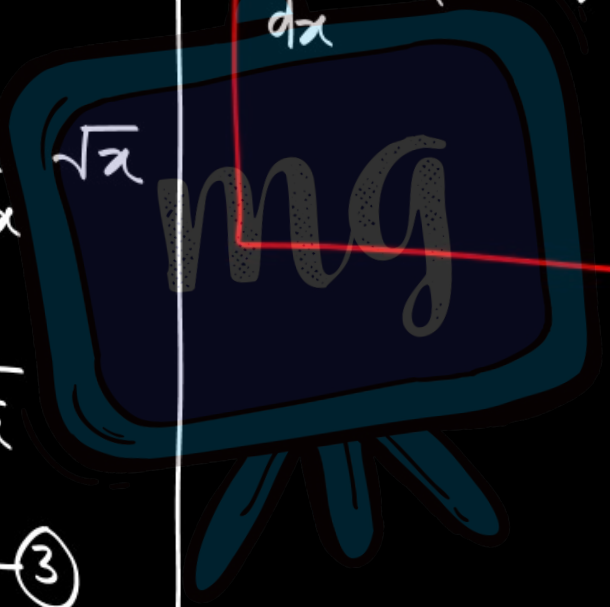
$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \frac{d}{dx}\sqrt{x}$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dv}{dx} = \frac{1}{2\sqrt{x(1-x)}} \quad \text{—— ③}$$

अब, समी. ①, ② व ③ से:

$$\frac{dy}{dx} = (\sin x)^x \left[x \cot x + \log \sin x\right] + \frac{1}{2\sqrt{x(1-x)}}$$



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9. $x^{\sin x} + (\sin x)^{\cos x}$

$y = x^{\sin x} + \sin x^{\cos x}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ——— ①

अब $u = x^{\sin x}$

$\log u = \log x^{\sin x}$

$\log u = \sin x \log x$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \sin x \times \frac{1}{x} + \log x \cdot \cos x$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \cdot \log x\right]$ ——— ②

इसीप्रकार,

$$v = \sin x^{\cos x}$$

$$\log v = \log \sin x^{\cos x}$$

$$\log v = \cos x \cdot \log \sin x$$

$$\frac{1}{v}\frac{dv}{dx} = \cos x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot (-\sin x)$$

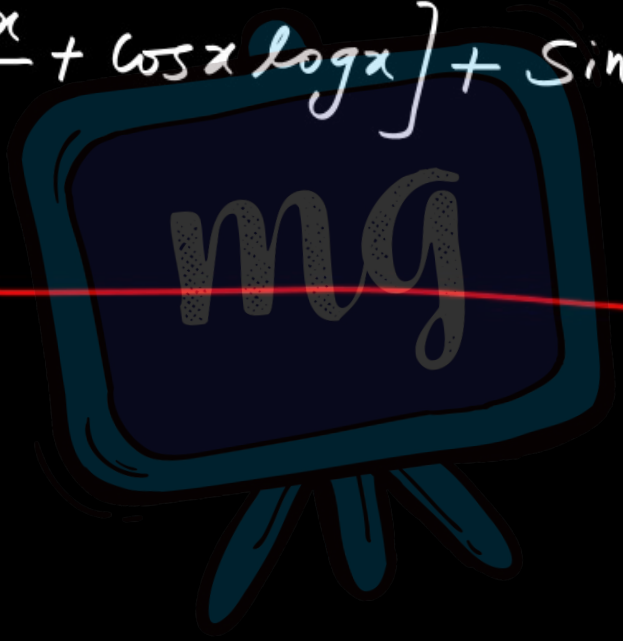
$$\Rightarrow \frac{dv}{dx} = \sin x^{\cos x}\left[\cos x \cdot \cot x - \sin x \log \sin x\right] \quad \text{——③}$$

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समी. ①, ② व ③ से :

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right] + \sin x^{\cos x}\left[\cos x \cdot \cot x - \sin x \cdot \log \sin x\right]$$



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प्रश्नावली 5.5

$$y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

$$y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{—— (1)}$$

अब, $u = x^{x\cos x}$

$$\log u = \log x^{x\cos x}$$

10. $x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

$$\Rightarrow \log u = x\cos x \log x$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\cos x \cdot \frac{1}{x} + \log x \frac{d}{dx}(x\cos x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x + \log x\left[x(-\sin x) + \cos x \cdot 1\right]$$

$$\Rightarrow \boxed{\frac{du}{dx} = x^{x\cos x}\left[\cos x + \log x\,(\cos x - x\sin x)\right]} \quad \text{— (2)}$$

इसीप्रकार, $v = \dfrac{x^2+1}{x^2-1}$

$$\frac{dv}{dx} = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$\frac{dv}{dx} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$$

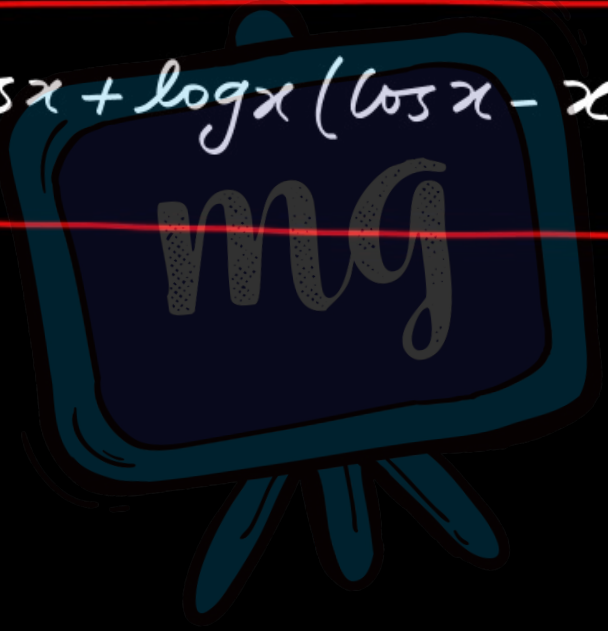
$$\boxed{\frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}} \quad \text{— (3)}$$

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अतः समी. ①, ② व ③ से:

$$\frac{dy}{dx} = x^{x\cos x}\left[\cos x + \log x(\cos x - x\sin x)\right] - \frac{4x}{(x^2-1)^2}$$



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प्रश्नावली 5.5

$y = (x\cos x)^x + (x\sin x)^{1/x}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ —— ①

अब, $u = (x\cos x)^x$

$\log u = \log(x\cos x)^x$

11. $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

$\Rightarrow \log u = x\log(x\cos x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{x\cdot\cos x}\frac{d}{dx}(x\cos x)$

$+ \log(x\cos x)\cdot 1$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{1}{\cos x}\left[x(-\sin x) + \cos x \cdot 1\right] + \log(x\cos x)$$

$$\Rightarrow \boxed{\frac{du}{dx} = (x\cos x)^x\left[\frac{1}{\cos x}(\cos x - x\sin x) + \log(x\cos x)\right]} \quad \text{---(2)}$$

हल कीजिए,

$$u = (x\sin x)^{1/x}$$

$$\log u = \log(x\sin x)^{1/x}$$

$$\Rightarrow \log u = \frac{1}{x}\log(x\sin x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{1}{x}\cdot\frac{1}{x\sin x}\frac{d}{dx}(x\sin x) + \log(x\sin x)\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{1}{x^2\sin x}\left[x\cdot\cos x + \sin x\cdot 1\right] + \log(x\sin x)\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \boxed{\frac{dv}{dx} = (x\sin x)^{1/x}\left[\frac{1}{x^2\sin x}(\sin x + x\cos x) - \frac{\log(x\sin x)}{x^2}\right]}$$

(3)

अतः, $$\boxed{\frac{dy}{dx} = (x\cos x)^x\left[\frac{1}{\cos x}(\cos x - x\sin x) + \log(x\cos x)\right] + (x\sin x)^{1/x}\left[\frac{1}{x^2\sin x}(\sin x + x\cos x) - \frac{\log(x\sin x)}{x^2}\right]}$$

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प्रश्नावली 5.5

$x^y + y^x = 1$

$u + v = 1$

$\frac{du}{dx} + \frac{dv}{dx} = 0$ ——①

अब, $u = x^y$

$\log u = \log x^y$

$\log u = y \log x$

12 से 15 तक के प्रश्नों में प्रदत्त फलनों के लिए $\frac{dy}{dx}$ ज्ञात कीजिए:

12. $x^y + y^x = 1$

$\frac{1}{u}\frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$ ——②

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इसीप्रकार,

$v = y^x$

$\log v = \log y^x$

$\log v = x \log y$

$\frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y \cdot 1$

$\Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$ ——③

दत्त समी. ①, ② व ③ से :

$x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$

$\Rightarrow x^y \cdot \frac{y}{x} + x^y \log x \frac{dy}{dx} + y^x \cdot \frac{x}{y}\frac{dy}{dx} + y^x \log y = 0$

$$\Rightarrow x^y \log x \frac{dy}{dx} + y^x \cdot \frac{x}{y} \frac{dy}{dx} = -x^y \cdot \frac{y}{x} - y^x \log y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-x^y \cdot \frac{y}{x} - y^x \log y}{x^y \log x + y^x \cdot \frac{x}{y}}}$$