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कक्षा – 12

गणित

अवकलज के अनुप्रयोग

अध्याय – 6

भाग - 9

प्रश्नावली 6.3

By : Rajendra Yadav Sir

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Q3. निम्नलिखित फलनों के स्थानीय उच्चतम या निम्नतम, यदि कोई हों तो, ज्ञात कीजिए तथा स्थानीय उच्चतम या स्थानीय निम्नतम मान, जैसी स्थिति हो, भी ज्ञात कीजिए।

(v) $f(x) = x^3 - 6x^2 + 9x + 15$

$f(x) = x^3 - 6x^2 + 9x + 15$

$\Rightarrow f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x^2 - 3x - x + 3)$

$= 3[x(x-3) - 1(x-3)]$

$f'(x) = 3(x-3)(x-1)$

अब, $f'(x) = 0$

$\Rightarrow 3(x-3)(x-1) = 0$

या तो $x-3=0$ या फिर $x-1=0$

$\Rightarrow x=3$ $\Rightarrow x=1$

अब, $f'(x) = 3x^2 - 12x + 9$

$\Rightarrow f''(x) = 6x - 12$

अब, $x=3$ के लिए:

$f''(3) = 6\times3 - 12 = 18 - 12 = 6 > 0$

$\Rightarrow$ स्थानीय निम्नतम मान होगा।

$\Rightarrow f(x) = x^3 - 6x^2 + 9x + 15$

$f(3) = (3)^3 - 6(3)^2 + 9\times3 + 15$

$= 27 - 54 + 27 + 15$

$f(3) = 15$

इसीप्रकार, $x = 1$ के लिए :

$$f''(x) = 6x - 12$$

$$\Rightarrow f''(1) = 6 \times 1 - 12 = 6 - 12 = -6 < 0$$

स्थानीय उच्चतम मान

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\Rightarrow f(1) = (1)^3 - 6(1)^2 + 9 \times 1 + 15$$

$$= 1 - 6 + 9 + 15 = 25 - 6 = 19$$

$$\boxed{f(1) = 19}$$

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(vi) $g(x) = \frac{x}{2} + \frac{2}{x}, x > 0$

$g(x) = \frac{x}{2} + \frac{2}{x}$

$\Rightarrow g'(x) = \frac{1}{2} - \frac{2}{x^2}$

$g'(x) = \frac{x^2 - 4}{2x^2}$

अब, $g'(x) = \frac{x^2 - 4}{2x^2} = 0$

$\Rightarrow x^2 - 4 = 0$

$x^2 = 4$

$\Rightarrow x = \pm\sqrt{4}$

$\Rightarrow \boxed{x = \pm 2}$

परन्तु $x > 0$

$\Rightarrow \boxed{x = 2}$

अब, $g'(x) = \frac{1}{2} - \frac{2}{x^2}$

$\Rightarrow g'(x) = \frac{1}{2} - 2x^{-2}$

अब, $g''(x) = 0 - 2(-2x^{-2-1})$

$\Rightarrow g''(x) = 4x^{-3}$

$\Rightarrow g''(x) = \frac{4}{x^3}$

अब, $x = 2$ के लिए :

$$g''(x) = \frac{4}{x^3}$$

$$\Rightarrow g''(2) = \frac{4}{(2)^3} = \frac{4}{8_2} = \frac{1}{2} > 0$$

अतः स्थानीय निम्नतम मान प्राप्त होगा।

अतः $g(x) = \frac{x}{2} + \frac{2}{x}$

$$\Rightarrow g(2) = \frac{2}{2} + \frac{2}{2}$$

$$g(2) = 1 + 1 = 2$$

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(vii) $g(x) = \dfrac{1}{x^2+2}$

$g(x) = \dfrac{1}{x^2+2}$

$\Rightarrow g'(x) = \dfrac{(x^2+2)\cdot 0 - 1(2x)}{(x^2+2)^2}$

$\Rightarrow g'(x) = \dfrac{-2x}{(x^2+2)^2}$

अब, $g'(x) = \dfrac{-2x}{(x^2+2)^2} = 0$

$\Rightarrow x = 0$

अब, $g'(x) = \dfrac{-2x}{(x^2+2)^2}$

$\Rightarrow g''(x) = \dfrac{(x^2+2)^2(-2) + 2x \cdot 2(x^2+2)\cdot 2x}{(x^2+2)^4}$

$$\Rightarrow g''(x) = \frac{-2(x^2+2)^2 + 8x(x^2+2)}{(x^2+2)^2}$$

$$\Rightarrow g''(x) = \frac{(x^2+2)\left[-2(x^2+2) + 8x\right]}{(x^2+2)^2}$$

$$\Rightarrow g''(x) = \frac{8x - 2(x^2+2)}{x^2+2}$$

अतः, $x = 0$ पर,

$$g''(0) = \frac{0 - 2(0+2)}{0+2} = \frac{-4}{2}$$

$g''(0) = -2 < 0$

$\Rightarrow$ स्थानीय उच्चतम मान होगा।

अब, $g(x) = \frac{1}{x^2+2}$

$\Rightarrow g(0) = \frac{1}{0+2}$

$\Rightarrow g(0) = \frac{1}{2}$



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(viii) $f(x) = x\sqrt{1-x}$ $0 < x < 1$

$$f(x) = x\sqrt{1-x}$$

$$f'(x) = x \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1) + \sqrt{1-x} \cdot 1$$

$$f'(x) = \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$$

$$f'(x) = \frac{-x + 2(1-x)}{2\sqrt{1-x}}$$

$$f'(x) = \frac{-x + 2 - 2x}{2\sqrt{1-x}}$$

$$f'(x) = \frac{-3x + 2}{2\sqrt{1-x}}$$

अब, $f'(x) = 0$

$\Rightarrow -3x + 2 = 0$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

अतः, $f'(x) = \dfrac{-3x+2}{2\sqrt{1-x}}$

$\Rightarrow f''(x) = \dfrac{2\sqrt{1-x}\,(-3) + (-3x+2)\cdot 2\cdot \dfrac{1}{2\sqrt{1-x}}}{4(1-x)}$

$\Rightarrow f''(x) = \dfrac{-6\sqrt{1-x} + \dfrac{(-3x+2)}{\sqrt{1-x}}}{4(1-x)}$

$$f''(x) = \frac{\frac{-6(1-x)+(-3x+2)}{\sqrt{1-x}}}{4(1-x)}$$

$$f''(x) = \frac{-6+6x-3x+2}{4(1-x)^{3/2}}$$

$$f''(x) = \frac{3x-4}{4(1-x)^{3/2}}$$

$$f''\left(\frac{2}{3}\right) = \frac{3\times\frac{2}{3}-4}{4\left(1-\frac{2}{3}\right)^{3/2}}$$

$$f''\left(\frac{2}{3}\right) = \frac{2-4}{4\left(\frac{3-2}{3}\right)^{3/2}}$$

$$f''\left(\frac{2}{3}\right) = \frac{-2}{4\left(\frac{1}{3}\right)^{3/2}} < 0$$

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अतः $f(x) = x\sqrt{1-x}$

$\Rightarrow f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1-\frac{2}{3}}$

$= \frac{2}{3}\sqrt{\frac{3-2}{3}}$

$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{\frac{1}{3}}$ उच्चतम मान